

UNEMPLOYMENT VOLATILITY  
&  
INVESTMENT IN SKILL VARIETY

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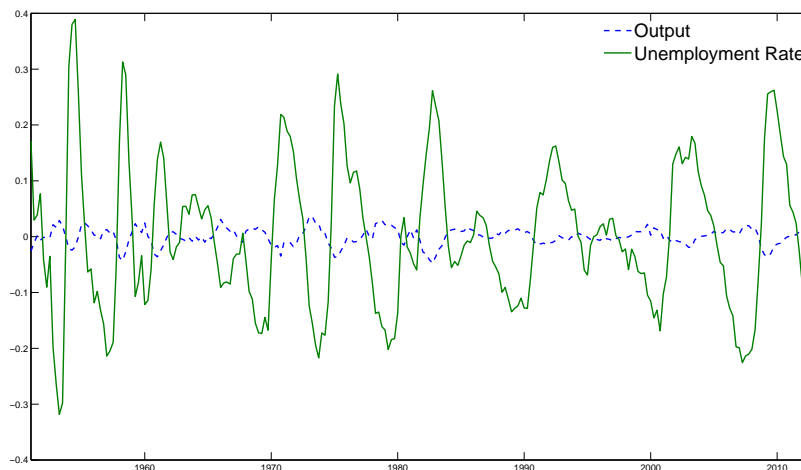
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*Still Preliminary*

**Abstract**

In the US, at business cycle frequencies, the standard deviation of unemployment is nine times higher than that of output. But the canonical search model generates a standard deviation of unemployment lower than that of output. Motivated by this *unemployment volatility* puzzle, we propose a new perspective on the labor market. We introduce unemployment into a real business cycle model by assuming that: (i) Workers differ in their skills; (ii) firms love variety of skills. Our model delivers a strong propagation mechanism and generates a standard deviation of unemployment in line with data.

Figure 1: Unemployment and Output



Note: Quarterly data (1951:1–2012:4), in logs, filtered using the H-P filter with a smoothing parameter of 1,600.

## 1. Introduction

Figure 1 displays postwar US output and unemployment rate at business cycle frequencies: Relative to output, unemployment is rather volatile. The standard deviation of unemployment is nine times higher than that of output. But the canonical search model, the workhorse model of the labor market, generates a standard deviation of unemployment lower than that of output (Robert Shimer, 2005). Motivated by this *unemployment volatility puzzle*, in a novel way, we introduce unemployment into a real business cycle (RBC) model. Our model generates a standard deviation of unemployment (and output) in line with data.

We study unemployment oscillations in a model where workers differ in their skills and firms love variety of workers' skills. In this extension of an RBC model, unemployment emerges because of two reasons: (i) as the job separation rate plays a minor role in US unemployment oscillations (Robert E Hall (2005a) and Robert Shimer (2012)), we follow much of the search and matching literature and assume that a constant share of employed workers lose their jobs every period; and (ii) we assume that job creation is costly.

The costs of job creation are supported by the household: Each period, the household decides the number of workers that are coordinated with firms. By creating jobs, households benefit in two ways. Firstly, a new job implies that an additional worker is employed and generates labor income. Secondly, because workers differ in their skills, the household exploits monopolistic profits from each employed worker by setting a markup over the marginal rate of substitution of consumption for leisure. The house-

hold creates jobs until it balances the costs and benefits of job creation.

Our model generates highly volatile unemployment. Unemployment oscillations result from changes in incentives and costs of job creation. In our model, a positive productivity shock increases wages because of labor demand pressure. Higher wages, in turn, increase the incentives for job creation: the household exploits a larger economic surplus of creating an additional job after a positive productivity shock. As a result, households create further jobs. This is not the case in search and matching models. In these models, the increase of wages in the aftermath of a positive productivity shock discourages firms to create jobs. In our model, because households create jobs, the increase in wages encourages job creation.

Our model has an additional endogenous amplification mechanism. In equilibrium, our model exhibits increasing returns to scale to specialization: intuitively, a higher employment rate enables workers to specialize in specific tasks that are specific to their skills, enhancing productivity. Thus, when in the aftermath of a positive productivity shock, employment increases, firms have further incentives to hire workers. This amplifies the effects explained above: it increases upward pressure in wages, further motivating households to create jobs.

Our model also outperforms the canonical RBC model, and solves two of its main problems: the lack of output volatility and the lack of output persistence (Robert G. King and Sergio T. Rebelo (1999)). The high unemployment volatility generated by our model propagates to output, increasing output volatility. This is achieved even without a Frisch elasticity inconsistent with micro studies.<sup>1</sup> Our extension of an RBC model is also able to increase persistence of output: even after 25 years past a productivity shock, the shock still has visible effects on output; yet, in the canonical RBC model, the shock has a neglectable effect 20 years past its emergence.

The paper is structured as follows. In Section 2., we review the literature on the unemployment volatility puzzle. We proceed with Section 3. by showing our modeling assumptions and with Section 4. by calibrating our model. In Section 5., we show how our model solves the unemployment volatility puzzle, and how it outperforms the canonical rbc model for both output's volatility and persistence. In Section 6., we conclude.

## 2. The Unemployment Volatility Puzzle Literature

Shimer (2005) shows that the unemployment volatility in the canonical search model is 20 times smaller than in data (18 times smaller in the search model used in this paper). The difference in results between our model and search models reflect differences in the job creation process. In the canonical search model, unemployed workers search for jobs every period, while firms open vacancies. Opening vacancies is costly: Firms contrast the expected value of a vacancy—which depends on the value of a filled job and the likelihood of filling a vacancy—with its costs. Furthermore, a matching function de-

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<sup>1</sup>For more details on the difference between micro and macro labor supply elasticities, see Raj Chetty, Adam Guren, Day Manoli and Andrea Weber (2011).

termines the number of matches. This matching function is homogeneous of degree one and concave, and determines the number of matches as a function of unemployed workers and vacancies. A job is created if, upon meeting, both worker and firm agree on a wage.

The unemployment volatility puzzle of search models results from (i) the interaction of congestion externalities (concave matching function) with vacancy posting costs and from (ii) the wage setting mechanism. A positive productivity shock induces firms to open more vacancies. But, as the number of vacancies increases, it is harder to fill each vacancy and, thus, more costly to hire a worker. At the same time, because an unemployed worker can more easily find a job, wages increase. These two effects together pin down the effect of higher labor productivity on unemployment.

Many researchers, all remaining in the framework of search models, have tried to solve the unemployment volatility puzzle. [Robert E. Hall \(2005b\)](#) and [Robert Shimer \(2004\)](#) advocate that real wage rigidities can help solve the unemployment volatility puzzle by removing the effect of productivity shocks on wages. [Robert E. Hall and Paul R. Milgrom \(2008\)](#) advocate an alternative wage bargain with a threat to keep bargaining next period. With this assumption, wages become less sensitive to market conditions.

Solutions advocating real wage rigidity, however, have been criticised on empirical grounds. In search models, job creation is determined by the wage of new hires. And [Christopher A. Pissarides \(2009\)](#), [Christian Haefke, Marcus Sonntag and Thijs van Rens \(2008\)](#), and [Anabela Carneiro, Paulo Guimarães and Pedro Portugal \(2012\)](#) found no evidence that the wages of new hires are rigid; they concluded that the wages of new hires are as cyclical as labor productivity.

[Marcus Hagedorn and Iouri Manovskii \(2008\)](#), on the other hand, advocate changes to the conventional calibration of search models. They advocate that the workers' bargaining power and the workers' surplus achieved by moving from unemployment to employment should be close to null. This calibration strategy implies a lower firms' surplus of hiring a new worker, increasing the sensitivity (in percentage terms) to productivity. Although this calibration strategy does increase the volatility of the canonical search model, it has also been criticised on the empirical validity of their assumptions: [James S Costain and Michael Reiter \(2008\)](#) show that by simply calibrating the canonical search model, it is impossible to obtain both a reasonable response to productivity shocks and a reasonable response to policy variables. Thus, by obtaining one, [Hagedorn and Manovskii \(2008\)](#) neglect the other.

In this paper, we do not try to solve the unemployment volatility puzzle. Instead, we propose an alternative method to model unemployment in an RBC model. And, in particular, we propose an alternative model that has a behavior consistent with data for two main variables: unemployment and output.

### 3. The Model

Our model is a simple extension of an RBC model. There are two agents in our model, firms and households. Firms produce the final good by employing capital,  $k_t$ , and a labor composite of all available labor types,  $l_t$ . Households consume the final good, rent capital, and supply labor. The labor supply in our model differs from the canonical RBC model. In the canonical RBC model, workers are homogeneous and the household only supplies labor through the intensive margin of labor. In our model, households supply workers with differentiated skills and through both the intensive and extensive margins of labor. Each period, the household decides hours per worker and the number of new jobs,  $x_t$ . The number of new jobs is determined by the costs of job creation and by the economic benefits of employing an additional worker. It is used as a control variable for employment,  $n_t$ , which follows the law of motion

$$n_t = (1 - \delta_n)n_{t-1} + x_t, \quad (1)$$

where  $\delta$  is a constant fraction of jobs that are exogenously destroyed each period.

#### 3.1. Representative Firm

Firms produce the final good, which can be used for consumption, investment, and the cost of job creation. To produce the final good, firms combine capital and labor in the form

$$y_t = a_t k_t^\alpha l_t^{1-\alpha}, \quad (2)$$

where  $\alpha$  is the capital share, and  $a_t$  is a common productivity factor. The labor input,  $l_t$ , is a composite of hours per worker,

$$l_t = \left[ \int_{j \in J} h_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \quad (3)$$

Each worker  $j$  supplies a differentiated skill, and if employed, works  $h_t(j)$  hours. Because of unemployment, at any given time  $t$ , only a subset of all labor types,  $J$ , is available. We denote this subset by  $J_t \subset J$ . Finally, the parameter  $\theta > 0$  governs the elasticity of substitution between differentiated labor types.

The first order conditions to the firm's cost minimization problem are

$$r_t = \alpha y_t / k_t, \quad (4)$$

$$W_t = (1 - \alpha) y_t / l_t, \quad (5)$$

$$W_t = \left[ \int_{j \in J_t} w_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \quad (6)$$

$$h_t(j) = \left( \frac{w_t(j)}{w_t} \right)^{-\theta} l_t, \quad (7)$$

where  $r_t$  is the rental rate of capital;  $W_t$  is the aggregate hourly wage; and  $w_t(j)$  is the hourly wage for the labor services of the worker  $j$ . These first order conditions are similar to the ones of RBC models extended with worker's differentiated skills (e.g. V. V.

Chari, Patrick J. Kehoe and Ellen R. McGrattan (2007)). The only difference lies on the fact that our model features unemployment, whereas other models assume full employment.

### 3.2. Representative Household

Households own the firms. Each household is composed of a large number of members of total measure unity who are all willing to work.<sup>2</sup> But, because of unemployment, a fraction  $n_t$  is employed and a fraction  $u_t = 1 - n_t$  is unemployed. We follow [Monika Merz \(1995\)](#) and assume that the household completely insures its members against employment risk, implying that members equally share consumption.

The household decides the number of jobs created each period,  $x_t$ . Job creation is an investment: It implies a cost at the time jobs are created,  $\phi(1 + \frac{1}{2}x_t)$ , but it also generates income in the following periods,  $w_t(j)h_t(j)$ . The only difference with respect to capital investment is that job creation has a direct effect on the disutility of labor,  $\frac{h_t(j)^{1+\psi^{-1}}}{1+\psi^{-1}}$ . We incorporate these changes to an otherwise standard household's problem:

$$\max_{c_t, h_t, k_{t+1}, n_t, x_t, w_t^h} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t - \int_0^{n_t} \chi \frac{h_t(j)^{1+\psi^{-1}}}{1+\psi^{-1}} dj \right]$$

subject to the budget constraint, labor demand, and the law of motion of employment

$$\begin{aligned} \int_0^{n_t} w_t(j)h_t(j)dj + (r_t + (1 - \delta_k))k_t &\geq c_t + k_{t+1} + \phi \left( x_t + \frac{1}{2}x_t^2 \right), \\ h_t(j) &\leq \left( \frac{w_t(j)}{w_t} \right)^{-\theta} l_t, \\ n_t &\leq (1 - \delta_n)n_{t-1} + x_t, \end{aligned}$$

where  $\beta$  is the discount factor;  $\chi$  is a measure of the disutility of labor;  $\psi$  is the Frisch elasticity of labor supply;  $\delta_k$  is the capital depreciation rate; and  $\phi$  is a scale parameter of the cost of job creation.

We anticipate symmetric equilibrium across labor types: All employed workers work the same number of hours,  $h_t$ , and earn the same wage,  $w_t^h$ . Then, the first order conditions are

$$w_t = \frac{\theta}{\theta - 1} \chi h_t^{\psi^{-1}} c_t \quad (8)$$

$$1 = \beta \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} (1 - \delta_k + r_{t+1}) \right]. \quad (9)$$

$$\phi(1 + x_t) = \frac{\theta + \psi}{\theta(\psi + 1)} h_t w_t + \beta(1 - \delta_n) \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} \phi(1 + x_{t+1}) \right]. \quad (10)$$

<sup>2</sup>Given our focus on the U.S. business cycle fluctuations, and the lack of cyclicity in the U.S. labor force, this assumption do not compromise our results ([Richard Rogerson and Robert Shimer, 2010](#)).

The first two equations are the same as in RBC models extended with worker's differentiated skills: Households set a constant mark-up over the marginal rate of substitution of consumption for labor, and invest in capital until they are indifferent between consuming one unit this period or investing and consuming it next period. The third equation, however, is one of the novelties of this paper. It is the *job creation equation*: The marginal cost of creating a job must equal the sum of (i) the marginal gain of an additional employed worker for the household in consumption units and (ii) the continuation value of employment.

### 3.3. Symmetric Equilibrium and Aggregation

In a symmetric equilibrium, Eqs. 3 and 6 imply<sup>3</sup>

$$l_t = h_t n_t^{\frac{\theta}{\theta-1}}, \quad (11)$$

$$W_t = w_t n_t^{\frac{1}{1-\theta}}. \quad (12)$$

These two equations are not standard in the RBC literature: The labor composite is not a linear function of employment, as well as the wage index is not equal to the hourly wage. But they are a natural result of two assumptions of our model. They result from the assumption of firms love-for-variety of skills and from the existence of unemployment. These assumptions generate increasing returns to scale in the form of labor division: As more workers are employed, the ability to distribute tasks that fit each skill is enhanced leading to higher productivity.<sup>4</sup>

Another aggregate consistency condition is the resource constraint:

$$y_t = c_t + k_{t+1} - (1 - \delta_k)k_t + \phi \left( x_t + \frac{1}{2}x_t^2 \right). \quad (13)$$

This equation states that final good production is devoted to consumption, investment, and costs of job creation. A competitive equilibrium for this economy is thus described by a set of allocations  $c_t, h_t, g_t, k_t, l_t, n_t, x_t, y_t$ , prices  $r_t, w_t, w_t^h$ , and common productivity factor  $a_t$  satisfying equations 4-13, given an exogenous process for the common productivity factor

$$a_t = \rho a_{t-1} + \varepsilon_t, \quad (14)$$

and the initial condition for employment, capital, and productivity:  $n_{-1}, k_{-1}$  and  $a_{-1}$ .  $\varepsilon_t$  is an i.i.d disturbance with mean zero.

## 4. Calibration

Our model is highly nonlinear. To proceed with the analysis, we log-linearize the model around the steady-state and calibrate it. We calibrate our model to the U.S. economy

<sup>3</sup>For details of obtaining these two equations see [Marc Melitz \(2003\)](#).

<sup>4</sup>Our results, however, are not only driven by this assumption.



and summarize our calibration in Table 1. We set each period  $t$  as a quarter. Accordingly, we follow much of the business cycle literature and set  $\beta = 0.99$ ,  $\delta_k = 0.025$ ,  $\alpha = 0.36$ ,  $\rho = 0.979$ , and  $var(\varepsilon) = 0.0072$ . We set  $\delta_n = 0.1$  to be consistent with the large flows out of employment in U.S. data. The choice over the Frisch labor supply elasticity,  $\psi$ , is controversial. Nevertheless, as this discussion goes beyond the scope of this paper, we set  $\psi = 1$  in our benchmark calibration, and assess the role of  $\psi$  in our sensitivity analysis. We follow the same strategy for  $\theta$  and set  $\theta = 6$  in our benchmark calibration. Finally, we use  $\phi$  and  $\chi$  to target a steady-state unemployment rate of 6% (approximately the unemployment rate in the U.S. in the postwar period) and to normalize hours worked to one.

## 5. Results

### 5.1. Second moments

In Table 2, we contrast the business cycle statistics of eight key macroeconomic variables estimated using U.S. data, with the ones generated by our model, by a search model, and by an RBC model. The U.S. data we use starts at 1951:1 and ends at 2012:4.<sup>5</sup> To generate the results of the three models, we assume that productivity shocks are the only driver of business cycles.<sup>6</sup> To produce all business cycle statistics, we use a Hodrick-Prescott smoothing parameter of 1600.

We conclude from Table 2 that our model delivers business cycle statistics close to the ones observed in the US economy. We also conclude that our model outperforms its two competitors in replicating US data. If we first consider unemployment, we confirm the unemployment volatility puzzle of search models: In data, the standard deviation of unemployment is 13.10, whereas the search model generates a standard deviation of 0.71. Our model, on the other hand, generates a standard deviation of unemployment of 12.43, very close to its data counterpart.

The unemployment volatility puzzle of search models, first documented by Shimer (2005), results from (i) the interaction of congestion externalities (concave matching function) with vacancy posting costs and from (ii) the wage setting mechanism. A positive productivity shock induces firms to open more vacancies. But the decrease in the vacancy filling ratio and the increase in wages pin down the effect of higher labor productivity on unemployment. Our model, however, does not have an unemployment volatility problem. Eq. 10 implies that an increase in wages encourages the households to create more jobs, thus increasing the sensitivity of unemployment to productivity shocks. Furthermore, firms' love for variety of skills generates increasing returns to scale in equilibrium which further contributes to higher volatility in our model.

Our extension of an RBC model contributes to solving one of the canonical RBC model's main problems: The lack of output volatility (see King and Rebelo (1999)). If we contrast the standard deviation of GDP generated by our model and generated by its competitors, we easily conclude that our model does a better job in replicating US

<sup>5</sup>GDP, consumption, and investment are real and in per capita terms. Consumption comprises both nondurables spending and services. Investment comprises both investment and durables spending.

<sup>6</sup>The two models are detailed in the appendix as well as the calibrations we used to obtain the results reported in Table 2.



data: The volatility of GDP in our model is 1.53, close to 1.51 in data, and far above the volatility of GDP generated by the RBC model and by the search model, 1.22 and 1.23. This is the case because the high unemployment volatility in our model amplifies the effect of productivity shocks on GDP. As for the volatility of total hours (other of the canonical RBC's main problems), our model, although not completely satisfying, does a better job in generating higher volatility: Our model generates about half of the volatility in total hours we observe in data. For the remaining variables, our model does a fair job even though it generates a smaller volatility for investment, partly due to the two types of investment in our model.

The models generate similar cross correlations with output. The exception is for the case of hours per worker. In our model, the cross correlation of hours per worker and output is below its data counterpart, while in search it is above. Regarding autocorrelations, except for wages per hour, our model generates more persistence of the effects of productivity shocks. This is particularly the case for the persistence of GDP, for which our model generates a auto-correlation of 0.77 close to 0.81 in data. Our model also generates more reasonable autocorrelations for unemployment and employment than the search model. For these two variables, the search model generates a staggering 0.25 autocorrelation, way below 0.88 and 0.89 (respectively) in data.<sup>7</sup>

## 5.2. Impulse Response Functions

To gain further intuition, we proceed by contrasting the impulse response functions (IRF) of our model with those of the RBC model and the search model (see Figure 2). The IRF confirm our main results from Table 2: In response to a productivity shock, (i) unemployment in our model is much more volatile than in the search model, and (ii) GDP in our model has a stronger and more persistent response than in both RBC and search models. One other interesting result is that the search model and the RBC model plots almost overlap each other: The search model is unable to amplify the effect of productivity shocks.

The response of unemployment and employment in our model is overwhelmingly higher than in the search model. Although unemployment and employment do respond in the search model, their response is insignificant near the response in our model: In the figure, it is almost a flat line over zero; our model, however, predicts that unemployment falls by 15% two and half years after the shock. Regarding GDP, both the impact and the persistence are higher in our model. In our model, GDP is clearly above trend even after 25 years after the productivity. In the RBC and search models, only 20 years past the emergence of the productivity shock, the GDP is close to trend.

In our model, total hours increase significantly in response to the productivity shock even though hours per worker becomes negative after one year. The response of consumption in our model is also much higher than in the search model and in the RBC model: In our model, consumption increases by 0.8% in its peak, whereas in the two competitors it increases by about 0.4%. The magnitude of response of wages and in-

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<sup>7</sup>This only holds with quarterly calibration. With monthly calibration, standard in the search literature, the autocorrelation of unemployment and employment in the search model are close to their data counterparts.

vestment do not differ significantly across the three models.

### 5.3. Sensitivity Analysis

The values assigned to some of our model's parameters are not standard in the literature. Therefore, we proceed with a sensitivity analysis of the results of our model. In Table 3, we display the standard deviation of GDP, and the standard deviations of the remaining seven variables relative to the one of GDP. We assess our the robustness of the results of our model to changes in the Frisch elasticity of labor supply,  $\psi$ , and in the elasticity of substitution between labor types,  $\theta$ .

A staggering result of our model is that the large volatility of unemployment relative to output is robust to changes in  $\psi$  and  $\theta$ : Our model does not have an unemployment volatility problem. The standard deviation of unemployment relative to the standard deviation of GDP is about 8, very close to the 8.68 in US data. The standard deviation of the remaining variables relative to the standard deviation of GDP are also quite robust to changes in  $\psi$  and  $\theta$ .

The standard deviation of output is, however, slightly sensitive to changes in  $\psi$  and  $\theta$ . In any case, it is always larger than the standard deviation of GDP in the search model and in the RBC model calibrated with the benchmark calibration. This suggests that our model has a strong amplification mechanism that is robust to changes in  $\psi$  and  $\theta$ . Even with a small Frisch elasticity, 0.2 consistent with micro studies (see [Chetty et al. \(2011\)](#)), our model is able to generate a standard deviation 10% higher than the RBC model with benchmark calibration.

## 6. Concluding Remarks

In this paper, our research agenda was to build a model that generates high unemployment volatility, as we observe in US data. For that, we extended an RBC model to feature unemployment in a novel way.

Our model is able to overcome the unemployment volatility problem of search problems ([Shimer \(2005\)](#)), generating unemployment volatility consistent with US data. Our model also outperforms the canonical RBC model by solving two of its main drawbacks: lack of output volatility and lack of output persistence ([King and Rebelo \(1999\)](#)).

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Table 1: Benchmark Parameters

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Discount factor:	$\beta = 0.99$
Rate of depreciation:	$\delta_k = 1.1^{1/4} - 1$
Rate of separation:	$\delta_n = 0.1$
Elasticity of substitution:	$\theta = 6$
Frisch elasticity:	$\psi = 1$
Capital share:	$\alpha = 0.36$
Weight in utility:	$\chi = 0.8896$
Cost parameters:	$\phi = 1.8997$
	$\gamma = 2$
Persistence of tech:	$\rho = 0.95$
Std of tech shock:	$\varepsilon = 0.0072$

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Table 2: Selected Moments

A. Standard Deviations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	1.51	13.10	0.84	0.52	1.75	0.89	6.17	0.85
Benchmark Model	1.53	12.43	0.76	0.40	0.90	0.66	2.65	0.43
Search Model	1.23	0.71	0.04	0.43	0.46	0.78	3.81	0.39
RBC Model	1.22	–	–	–	0.44	0.79	3.83	0.39
B. Relative Standard Deviations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	1.00	8.68	0.56	0.35	1.16	0.59	4.09	0.56
Benchmark Model	1.00	8.11	0.50	0.26	0.59	0.43	1.73	0.28
Search Model	1.00	0.57	0.04	0.35	0.37	0.64	3.09	0.32
RBC Model	1.00	–	–	–	0.36	0.65	3.13	0.32
C. Cross Correlations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	1.00	-0.79	0.80	0.71	0.84	0.17	0.88	0.75
Benchmark Model	1.00	-0.82	0.82	0.66	0.98	0.97	0.78	0.88
Search Model	1.00	-0.88	0.88	0.98	0.98	0.99	0.99	0.90
RBC Model	1.00	–	–	–	0.98	0.99	0.99	0.90
D. Autocorrelations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	0.81	0.89	0.90	0.81	0.89	0.73	0.84	0.78
Benchmark Model	0.77	0.93	0.93	0.70	0.84	0.65	0.94	0.82
Search Model	0.73	0.25	0.25	0.74	0.76	0.74	0.73	0.80
RBC Model	0.72	–	–	–	0.71	0.73	0.71	0.80

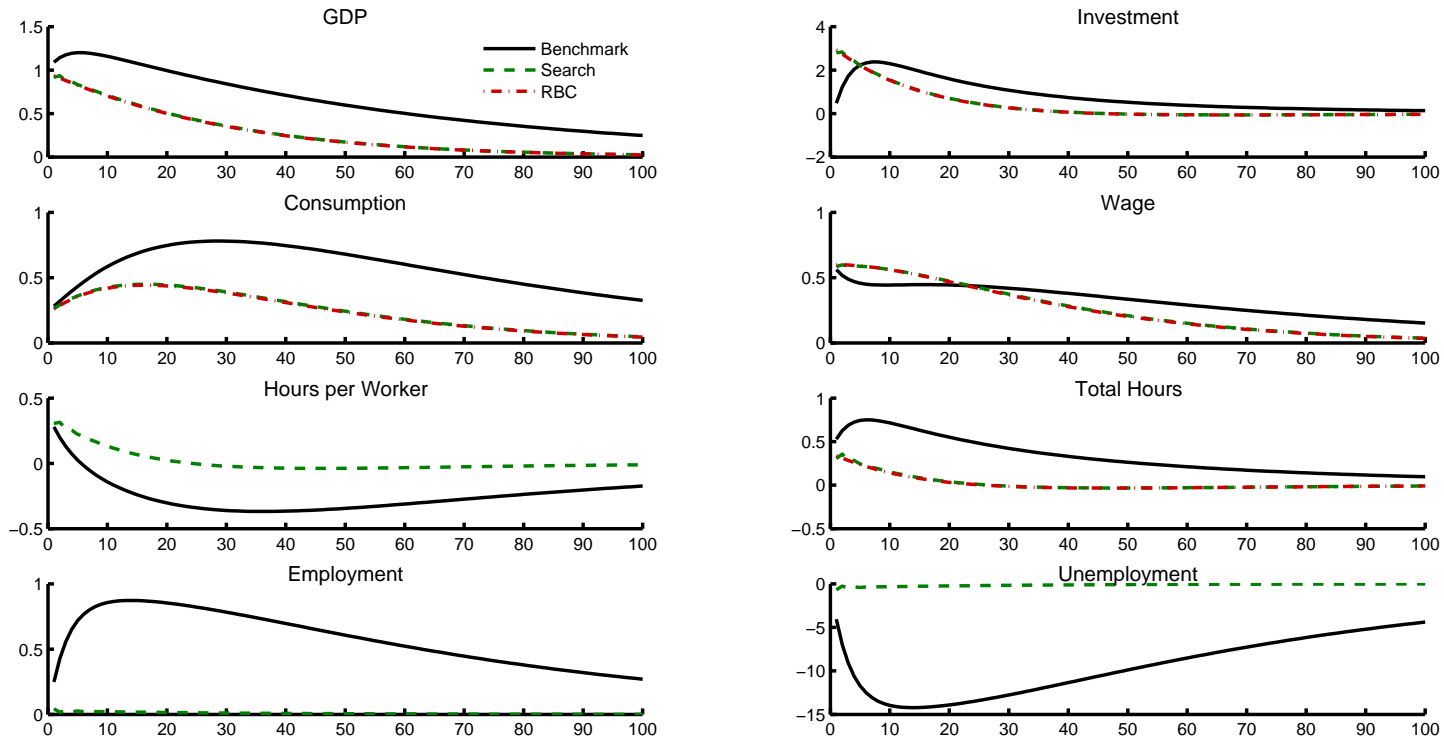
*Note:* This table shows the standard deviation, relative (to output) standard deviation, cross correlation with output, and autocorrelation of 8 variables of interest in data, our model, search model, and RBC model. The variables of interest are output, unemployment, employment, hours per worker, total hours, wage, investment, and consumption. It was used an hp filter smoothing parameter of 1600.

Table 3: Sensitivity Analysis

	Relative Standard Deviations							
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	1.51	8.68	0.56	0.35	1.16	0.59	4.09	0.56
Baseline Calibration	1.53	8.11	0.50	0.26	0.59	0.43	1.73	0.28
$\psi = 0.2$	1.32	8.04	0.49	0.09	0.51	0.58	1.81	0.28
$\psi = 0.5$	1.42	8.09	0.50	0.17	0.55	0.50	1.69	0.28
$\psi = 4$	1.78	8.05	0.49	0.43	0.68	0.33	2.19	0.28
$\theta = 3$	1.67	8.44	0.52	0.22	0.57	0.45	1.74	0.33
$\theta = 25$	1.46	7.97	0.49	0.27	0.59	0.43	1.80	0.27
$\theta = 100$	1.44	7.94	0.49	0.28	0.59	0.43	1.82	0.27

*Note:* This table shows the sensitivity of the results of our model to changes in the cost function parameter  $\gamma$ , in the Frisch elasticity of labor supply  $\psi$ , and in the elasticity of substitution between labor types  $\theta$ .

Figure 2: Impulse Response Functions



Note: This figure plots the impulse response function of our model (full line), of the search model (dashed line), and of the RBC model (dash dot line) to a 1% productivity shock.



## A Real Business Cycle Model

In this appendix, I explain the real business cycle model used in the paper.

### A1. Households

The household's utility maximization problem is the following:

$$\max_{c_t, h_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_{t+i} - \chi \frac{h_{t+i}^{1+\psi^{-1}}}{1+\psi^{-1}} \right]$$

subject to

$$d_t + w_t h_t + (r_t + (1 - \delta_k))k_t \geq c_t + k_{t+1},$$

The first order conditions are

$$w_t = \chi h_t^{\psi^{-1}} c_t \tag{A.1}$$

$$1 = \beta E_t \left[ \frac{c_t}{c_{t+1}} (1 - \delta_k + r_{t+1}) \right]. \tag{A.2}$$

### A2. Firms

The representative firm produces the final good using capital and labor:

$$y_t = a_t k_t^\alpha h_t^{1-\alpha}. \tag{A.3}$$

Cost minimization implies

$$r_t = \alpha y_t / k_t, \tag{A.4}$$

$$w_t = (1 - \alpha) y_t / h_t. \tag{A.5}$$

### A3. Equilibrium

To complete the model, we add the resource constraint

$$y_t = c_t + k_{t+1} - (1 - \delta_k)k_t. \tag{A.6}$$

#### A4. Calibration

We use  $\chi$  to target steady state hours:  $h = 1$ . We follow the real business cycle literature to calibrate the remaining parameters:  $\beta = 0.99$ ,  $\alpha = 0.36$ ,  $\delta_k = 1.1^{1/4} - 1$ , and  $\psi = 1$ . The parameters regarding the productivity shock are the same as for the main model of this paper.

## B Search and Matching Model

In this appendix, I explain the search and matching model used in the paper. We assume that the law of motion of employment is

$$n_{t+1} = (1 - \delta_n)n_t + m_t, \quad (\text{B.1})$$

where  $m_t$  is the number of new matches. The number of new matches is determined by a matching function which depends on two arguments, vacancies and unemployed workers:

$$m_t = \sigma(1 - n_t)^\eta v_t^{1-\eta}, \quad (\text{B.2})$$

where  $\sigma$  is a scale parameter;  $\eta$  is the elasticity of the matching function with respect to unemployment; and  $v_t$  is the number of vacancies. The job finding probability is

$$f(\theta_t) = \sigma\theta_t^{1-\eta}, \quad (\text{B.3})$$

while the job filling probability is

$$q(\theta_t) = \sigma\theta_t^{-\eta}, \quad (\text{B.4})$$

where  $\theta_t \equiv \frac{v_t}{1-n_t}$  is the labor market tightness.

#### B1. Households

Let  $V_t$  denote the household's lifetime utility. Taking as given the probability that the household's members find a job, the objective of the household is to choose a path for

$c_t$  that maximizes

$$V_t = \max_{\{c_t\}} \left[ \log c_t - \gamma n_t \frac{h_t^{1+\psi}}{1+\psi} + \beta E_t V_{t+1} \right], \quad (\text{B.5})$$

subject to

$$c_t = w_t n_t h_t + T_t + \Pi_t, \quad (\text{B.6})$$

$$n_{t+1} = (1 - \delta_n) n_t + f(\theta_t)(1 - n_t), \quad (\text{B.7})$$

where  $\Pi_t$  are profits stemming from owning the firms.

The maximization problem of the household implies that profits are discounted by

$$\Lambda_{t+1} \equiv \beta E_t \frac{c_t}{c_{t+1}}. \quad (\text{B.8})$$

## B2. Firms

Each firm employs  $n_t$  workers, and produces output,  $y_t$ , by means of the Cobb-Douglas production function

$$y_t = a_t k_t^\alpha (n_t h_t)^{1-\alpha}. \quad (\text{B.9})$$

Firms decide the number of vacancies to open each period,  $v_t$ . For each vacancy open, firms pay a cost  $\kappa$ . Firms have the ability to transform final goods into capital goods by means of a linear technology. Capital goods depreciate at a rate  $\delta_k$  per period. Let  $J_t$  denote the value of the representative firm. The firm's objective is to choose a path for  $v_t$  and for  $k_{t+1}$  that maximizes

$$J_t = \max_{v_t, k_{t+1}} \left( a_t k_t^\alpha (n_t h_t)^{1-\alpha} + (1 - \delta_k) k_t - k_{t+1} - w_t n_t h_t - v_t \kappa + E_t [\Lambda_{t+1} J_{t+1}] \right), \quad (\text{B.10})$$

subject to

$$n_{t+1} = (1 - \delta_n) n_t + q(\theta_t) v_t. \quad (\text{B.11})$$

Let  $J_n$  denote the marginal value of a worker to the firm. Then, the first-order con-

dition of the firm's maximization problem with respect to  $v_t$  can be expressed as

$$E_t [\Lambda_{t+1} J_{n,t+1}] q(\theta_t) = \kappa, \quad (\text{B.12})$$

while the envelop condition for employment can be expressed as

$$J_{n,t} = (1 - \alpha) \frac{y_t}{n_t} - w_t h_t + (1 - \delta_n) E_t [\Lambda_{t+1} J_{n,t+1}]. \quad (\text{B.13})$$

Let  $J_{k,t}$  denote the marginal value of capital to the firm at time  $t$ . Then, the first-order condition of the firm's maximization problem with respect to  $k_{t+1}$  can be expressed as

$$1 = E_t [\Lambda_{t+1} J_{k,t+1}], \quad (\text{B.14})$$

while the envelop condition for capital can be expressed as

$$J_{k,t} = \alpha \frac{y_t}{k_t} + 1 - \delta_k. \quad (\text{B.15})$$

We combine the last two equations by evaluating Eq. B.15 at  $t + 1$  and substituting it into Eq. B.14:

$$1 = E_t \left[ \Lambda_{t+1} \left( \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta_k \right) \right]. \quad (\text{B.16})$$

### B3. Wages and Hours per worker

We assume that wages and hours per worker are determined at the start of each period, and that they are the result of Nash bargaining between the worker and the firm:

$$w_t = \arg \max_{w,h} \tilde{V}_{n,t}(w,h)^\phi \tilde{J}_{n,t}(w,h)^{1-\phi}, \quad (\text{B.17})$$

where  $\tilde{V}_{n,t}(w,h)$  is the marginal value to the household of having a worker employed at some wage  $w$  per  $h$  hours worked rather than unemployed;  $\tilde{J}_{n,t}(w,h)$  is the marginal value to the firm of employing an additional worker at some wage  $w$  per  $h$  hours worked; and  $\phi$  is the worker's bargaining power.

Following the steps in [Shimer \(2012\)](#), we find that  $w_t$  and  $h_t$  satisfy the equations:

$$\phi J_{n,t} = w_t h_t (1 - \phi) - \gamma \frac{h_t^{1+\psi}}{1 + \psi} c_t (1 - \phi) + \phi (1 - \delta_n - f(\theta_t)) E_t [\Lambda_{t+1} J_{n,t+1}], \quad (\text{B.18})$$

$$\gamma h_t^{1+\psi} = (1 - \alpha) \frac{y_t}{n_t c_t}. \quad (\text{B.19})$$

#### B4. Equilibrium

To complete the model, we add the resource constraint

$$y_t = c_t + k_{t+1} - (1 - \delta_k) k_t + v_t \kappa. \quad (\text{B.20})$$

#### B5. Calibration

To calibrate the model, we use  $\kappa$ ,  $\sigma$ , and  $\gamma$  to target steady-state employment, hours per workers, and job filling probability. In particular, we target  $n = 0.9422$ ,  $h = 1$ , and  $q = 0.7$ . We calibrate the remaining parameters following the real business cycle literature and search and matching literature:  $\beta = 0.99$ ,  $\alpha = 0.36$ ,  $\delta_k = 1.1^{1/4} - 1$ ,  $\psi = 1$ ,  $\eta = 0.5$ ,  $\delta_n = 0.1$ , and  $\phi = 0.5$ . The parameters regarding the productivity shock are the same as for the main model of this paper.