How powerful are trade unions?
A skill-biased technological change approach

António Neto, Óscar Afonso and Sandra T. Silva *

(preliminary version, not to be cited)

Abstract

This paper proposes a new theoretical framework aiming to understand the link between technological change, skill premium and employment, combining a skill-biased technological change (SBTC) framework with a collective bargaining structure perspective. Our results suggest that in the presence of unions: (i) skill premium might be higher depending on the bargaining structure; (ii) it is possible to achieve higher employment and higher wages without increasing or even implying unemployment (iii) unions fail to anticipate their impact on the path of technological-knowledge bias.

Keywords: Skill-biased technological change; Bargaining structure; Unemployment; Economic growth.

JEL Codes: O4, J5, E24.

1 Introduction

Over the last decades the rising in income and wage inequality has been the rule in several advanced economies. According to Chusseau et al. (2008), “this development is now well documented and it may be observed (...) for workers with similar skills and between skilled and less skilled workers” (p. 411). Skill-biased technological change is claimed to be one of the most important explanations for this pattern. Violante (2008) defines it as “a shift in the production technology that favours skilled (...) labour over unskilled labour by increasing its relative productivity and, therefore, its relative demand. Ceteris paribus,
SBTC induces a rise in the skill premium - the ratio of skilled to unskilled wages”. On the other hand, Chusseau et al. (2008) states that SBTC “occurs when technical progress increases the total relative demand for skill of the economy for given prices of skilled labour $H$ and unskilled labour $L$”. (p. 412). Assuming that the relative supply of skilled labour is constant, this effect will result in (i) a higher skill premium and/or (ii) unskilled labour unemployment.

Nevertheless, what may explain SBTC in the first place? One of the most important explanations considers the possibility of an “endogenous” direction of technical change (Acemoglu (1998)). In other words, the amount of R&D activity conducted by firms might crucially influence the new type of arriving technology (i.e. “high” versus “low”), which in turn may determine how high(or low)-skilled workers are affected. Following Violante (2008), the main determinants that can endogenously influence the type of conducted R&D activity are market size, relative prices and institutions. Considering this last approach, we focus our analysis on the relationship between labour market institutions and skill-biased technological change, namely trade unions. Thus, there are several mechanisms by which unions can affect wages and, therefore, the type of research activity conducted by firms. In particular, (i) direct mechanisms, such as bargaining on behalf of covered workers to increase (or to maintain) wages, entailing costly investment in technology that is complementary with these workers; and (ii) indirect mechanisms, such as rise workers effort due to an increase in wages (see Bryson (2007)). In our paper we outline a detail analysis of the impacts of trade unions into SBTC through the first type of mechanism.

Figure 1 provides us a rather interesting picture. First, we calculated the average trade union density among the OECD countries and divided them in two groups, depending on whether their trade union density is above or below the average. Then, we compared their performance taking into consideration four different variables: a) decile $9^{th}/1^{st}$ ratio of gross earnings; b) relative number of high-tech enterprises; c) unemployment rate; and d) relative high-tech employment. As we can observe, countries with higher trade unions density (or, more precisely, above the average) seem to perform better in terms of high-tech technology, as depicted in Figures 1 (b,d), and have a lower unemployment rate, as suggested in Figure 1c). On the other hand, these countries seem also to have higher wage dispersion (Figure 1a), which is, at least, counter-intuitive, if one considers that trade unions essentially represent low-pay workers and try to negotiate higher wages on behalf of these covered workers.

The aforementioned data discussion provides us the motivation to ask the following question: what is the true relationship between unions and SBTC? Further, the literature combining SBTC with labour market imperfections is relatively scarce. In fact, Chang

1For a survey of the most important literature see Acemoglu (2002b), Aghion (2002), Hornstein et al. (2005), Chusseau et al. (2008) and Kurokawa (2014).
et al. (2007) and Lingens (2007, 2003) are the most relevant references on this topic.²

Figure 1
Notes: 1) The average trade union density may differ from graph to graph depending on the available information for each country. For example, if we have information for Portugal regarding the “decile ratio of gross earnings” but not for the “Relative high-tech employment”, the average trade union density would be different, since Portugal would be used on the first graph but not on the last one. 2) All calculations are available upon request.

According to Chusseau et al. (2008), p. 451:

A number of works have already explored the interactions between labour market institutions and labour market adjustment on the one hand, and globalization and technical change on the other hand. Kreickemeier and Nelson (2006) however remark that there is again much room for study in this respect. They suggest interactions between fair wage

²Lingens (2004)’ book, an intermediate analysis, is perhaps the first attempt to explain the relationship between skill premium and wage bargaining in labour market.
constraint and union bargaining, and their relation with NST and SBTC, as a potentially fruitful research programme.

Therefore, our motivation is straightforward according to three main reasons. Firstly, taking into account the recent economic developments worldwide, further insights into the relationship between wages and employment is crucial to promote an adequate economic policy towards a sustainable recovery (Blanchard (2007)). Secondly, in the specific context of the European Monetary Union (EMU) (single currency and a common inflation target for monetary policy) additional knowledge on the linkage between skill-biased technological change and employment under a collective bargaining structure perspective seems critical to assess the type of institutions, as well as the level of bargaining centralisation, that are more likely to ensure a steady economic growth rate. Finally, we try to fill the gap between the existent standard SBTC framework and the baseline labour market models by combining both approaches into a generic benchmark model. In particular, as a departing point from the SBTC models, we follow Afonso and Leite (2010), Afonso (2006), Acemoglu (2003) and Aghion and Howitt (1992); and from the labour market framework Dunlop (1944), Ross (1948) and McDonald and Solow (1981).

The paper proceeds as follows. Section 2 provides a general description of the model. Section 3 describes the equilibrium and its main properties. Section 4 provides a sensitivity analysis and Section 5 concludes.

2 The Model

2.1 Households

The economy is populated by a time-invariant number of heterogeneous individuals - continuously indexed by \(a \in [0,1]\) (Romer (2011) Chang et al. (2007) and Afonso (2006)). Each individual decides the allocation of income, which is partly lent in return for future interest, and part spent on consumption of the composite final good. To simplify, assume that an exogenous threshold individual \(\bar{a}\) exists, such that individuals \(a \leq \bar{a}\) are low-skilled and \(a > \bar{a}\) are high-skilled. With an inter-temporal elasticity of substitution \(1/\phi\), the individual’s utility with ability \(a\) depends positively on its consumption and negatively on the amount of labour it supplies as follows:

\[
U = \int_0^\infty \left[ \frac{c(a,t)^{1-\phi} - 1}{1 - \phi} - \frac{1}{\gamma} m(a,t) \right] e^{-\rho t} dt, \gamma > 1
\]  

subject to the following flow budget constraint:

\[
\dot{K}(a,t) = r(t)K(a,t) + W_m(t)m(a) - c(a,t)
\]
where:

\[
\begin{aligned}
m &= H & a > \bar{a} \\
m &= L & a \leq \bar{a}
\end{aligned}
\]

\(c(a, t)\) is the amount of consumption by the individual with the ability \(a\) at a time \(t\) of the final good; \(\rho > 0\) is the homogeneous subjective discount rate; \(W_m\) is the price paid for a unit of \(m\)-type labour. Finally, the standard Euler equation is given by:

\[
\hat{c} = \frac{\dot{c}(a, t)}{c(a, t)} = \frac{r(t) - \rho(t)}{\phi}.
\]

The household’s only other choice variable is its labour supply. Since its spending equals \(r(t)K(a, t) + w_m(t)m(a)\), the problem for choosing \(L\) can be written as follows:

\[\max_{m(a)} r(t)K(a, t) + W_m(t)m(a) - \frac{1}{\gamma}m(a)^{\gamma}\]

We can now get the labour supply curve for the low- and high-skilled workers, respectively.

\[
\begin{aligned}
L^S &= W_L^{-\frac{1}{1-\gamma}} \\
H^S &= W_H^{-\frac{1}{1-\gamma}}
\end{aligned}
\]

### 2.2 Final goods sector - firms, trade unions and bargaining structure

#### 2.2.1 Firms

Following the contributions of [Afonso (2006)] and [Acemoglu and Zilibotti (2001)], firms are indexed by \(n\) over the range \([0, 1]\). There are two substitute production technologies: a high(low)-technology that uses a combination of high(low)-specific quality adjusted intermediate goods indexed by \(j \in [J, 1]\) \((j \in [0, J])\) and high(low)-skilled labour. The production function of firm \(n\) at time \(t\) is given by:

\[
Y_n(t) = A \left\{ \left[ \int_0^J \left( q^{(j, t)} x_n(k, j, t) \right)^\alpha dj \right] \cdot (1 - n) L_n \right\}^\beta + \left[ \int_0^J \left( q^{(j, t)} x_n(k, j, t) \right)^\alpha dj \right] \cdot [n H_n]^{\beta} \right\}
\]

Parameter \(A\) is an exogenous and positive variable representing the productivity level, which depends on several factors, such as the country’s institutions - see, e.g., [Acemoglu and Zilibotti (2001)]. \(x_n(k, j, t)\) corresponds to the quantity of \(j\) used by firm \(n\), whereas \(3\)Since it is not our purpose to analyse the role of the government, we consider that the unemployed workers receive an exogenous amount of unemployment benefits. For further details please see [Chang et al. (2007)].
$q^{k(j,t)}$ measures its quality level, with $q > 1$ and $k$ as the highest quality rung at time $t$ (Aghion and Howitt (1992)). Parameter $\alpha \in [0, 1]$ and $\beta \in [0, 1]$ are the intermediate goods and labour share in production, respectively. Following Chang et al. (2007), we assume decreasing returns to scale, i.e., $0 < \alpha + \beta < 1$. This can be justified by the existence of other fixed factors (e.g., land). $h > l \geq 1$ captures for an absolute productivity advantage of $H$ over $L$. On the other hand, $n$ and $(1 - n)$ account for a relative productivity advantage of either type of labour, implying therefore that $L$ is relatively more productive in final goods indexed by smaller $n$s, and vice-versa. Given the production technology 4, the representative firm attempts to maximize its profit $\Pi(t)$ as follows:

$$\Pi(t) = P_n(t)Y_n(t) - W_L(t)L_n(t) - W_H(t)H_n(t) - \int_0^J x_{jn}(t)P_j(t)dj - \int_J^1 x_{jn}(t)P_j(t)dj$$  \hspace{1cm} (5)$$

$P_n(t)$ corresponds to the final good price, $W_L(t)$ and $W_H(t)$ to the wages of low-skilled and high-skilled labour, and $P_j(t)_{[0,J]}$ and $P_j(t)_{[J,1]}$ to the prices of intermediate goods indexed by $j \in [0,J]$ and $j \in [J,1]$, respectively. Moreover, the demand of final goods firms for intermediate goods can be obtained by $\frac{\partial x_n}{\partial X_{(k,j,t)}^{[0,J]}} = 0$ and it is described as follows:

$$x_n(k, j, t)|_{j \in [0,J]} = P_j^{-\frac{1}{1-\alpha}} (\alpha AP_n)^{\frac{1}{1-\alpha}} [(1 - n) \cdot L_n]^{\frac{\beta}{1-\alpha}} (q^{k(j,t)})^{\frac{\alpha}{1-\alpha}}$$  \hspace{1cm} (6)$$

$$x_n(k, j, t)|_{j \in [J,1]} = P_j^{-\frac{1}{1-\alpha}} (\alpha AP_n)^{\frac{1}{1-\alpha}} [nhH_n]^{\frac{\beta}{1-\alpha}} (q^{k(j,t)})^{\frac{\alpha}{1-\alpha}}$$  \hspace{1cm} (7)$$

Taking into consideration the mix evidence regarding how trade unions actually work and their impact on wages and SBTC (Gartner et al. (2013); Bryson (2007)), we analyse three types of situations in the labour market, namely the standard perfect competition and two types of trade unions: a monopoly union (Dunlop (1944); Kaufman (2002)) and a managerial trade union (McDonald and Solow (1981); Ross (1948); Chang et al. (2007)). In the last two cases, (i) all low-skilled workers are members of the trade union and (ii) the trade union does not bargaining over the high-skilled wage.

2.2.2 Perfect competition

In this case, wages of the low- and high-skilled workers are equal to their marginal productivity, as follows:

$$\frac{\partial P_n Y_n}{\partial L_n} = PML = W_L$$

$$\frac{\partial P_n Y_n}{\partial H_n} = PMH = W_H$$

Therefore, recalling 4 and taking into account 6 and 7 one can get the marginal
productivity of labour:

\[ W_L = \beta L_n^{-1} [(1 - n) IL_n]^\beta .P_n(t).A. \left[ \int_0^1 (q^{(j,t)}x_n (k, j, t))^\alpha \, dj \right] = \beta L^{-1} Y_L \quad (8) \]

\[ W_H = \beta H_n^{-1} [nhH_n]^\beta .P_n(t).A. \left[ \int_0^1 (q^{(j,t)}x_n (k, j, t))^\alpha \, dj \right] = \beta H^{-1} Y_H \quad (9) \]

where: \( Y_L (Y_H) \) could be seen as the contribution of the low-skill (high-skill) components, \textit{i.e.} technology and workers, to the production function. Figure 2 illustrates graphically the labour market in perfect competition. Notice that the equilibrium high-skilled wage is higher than the equilibrium low-skilled wage. This relates with the fact that high-skilled workers are, in absolute terms, more productive than low-skilled workers.

![Figure 2: Labour market - Perfect Competition](image)

2.2.3 Trade unions - Monopoly Union

The monopoly trade union framework was first proposed by \textit{Dunlop (1944)} and \textit{Ross (1948)}. Within this set-up, the union decides unilaterally the level of wages, leaving firms to choose the level of employment afterwards. The monopoly trade union’s utility function has the following Stone-Geary:

\[ U = (W_L(t) - \overline{W}_L)^{1-v} \left( L_n^D \right)^v \quad (10) \]

where \( W_L(t) \) corresponds to the low-skilled wage and \( \overline{W}_L \) to the perfect competition low-skilled wage. \( L_n^D \) corresponds to firm’s demand curve for workers.\(^4\) The value of \( v \) states whether the union is employment-oriented or wage-oriented. Since wages are fixed by

\(^4\)For a survey of the literature see \textit{Lawson (2011)} and \textit{Kaufman (2002)}.

\(^5\)Since unions do not have any preferences regarding the level of investment, we avoid the problem of underinvestment. For further information see \textit{Clark (1990)} and \textit{Booth (1993)}.
unions before employment, they can anticipate the impact of wages on the employment level. $L_n^D$ is given by the following equation and can be obtained by $\frac{\partial P_n}{\partial L_n} = W_L(t)$:

$$L_n^D = W_L(t) \left( \beta P_n A \left( \int_0^J \left( q^{k(j,t)} x_n(k,j,t) \right)^{\alpha} dx \right) \right)^{\frac{1}{1-\beta}} [(1-n) l]^{1-\beta}$$

(11)

Therefore, replacing (11) into (10) and maximizing it in order to $W_L(t)$ taking into account $6$ we get the standard monopoly wage as a mark-up over the perfect competition wage.

$$W_M^L = \left( 1 - \left( 1 - v \right)^{\beta} \right) W_L$$

(12)

**Result 1:** $W_M^L > \overline{W_L}$.

**Proof.** Since $v < 1$ and $\beta < 1$, $\left( \frac{1-(1-v)^\beta}{v} \right) > 1$

Figure 3 illustrates the monopoly union case, *ceteris paribus*. This implies that the high-skilled wage as well as the available technology is the same. This might not be the case, as we will see in the following sections.

**2.2.4 Trade unions - Efficient Bargaining**

The efficient bargaining model was first proposed by [McDonald and Solow (1981)](https://doi.org/10.1086/260955). In this case, through a generalized Nash bargaining problem, both union and firms negotiated over wages and employment, taking into consideration the demand of final goods firms’ for intermediate goods.\(^6\) The optimization problem can be written as:

$$\max \Omega = (U - \overline{U})^\theta (\Pi - \overline{\Pi})^{1-\theta}$$

(13)

\(^6\)For a critical review see [Layard and Nickell (1990)](https://doi.org/10.1086/260955).

\(^7\)Again, since unions do not have any preferences regarding the level of investment, we avoid the problem of underinvestment.
s.t.: \( x_i = \text{argmax}_{x_j} \Pi \)

where \( \theta \in [0, 1] \) is the bargaining power of the union. \( \bar{\Pi} \) and \( \bar{U} \) are the disagreement point of the final good firm and union, respectively. We assume that, without reaching an agreement, the employment level regarding the low-skilled workers would be zero and, therefore, \( \bar{U} = 0 \) and \( \bar{\Pi} = \Pi_H \) where \( \Pi_H \) corresponds to the firm’s profit using only high-skilled workers and high-specific quality adjusted intermediate goods.

Therefore, by maximization of equation (13) with respect to \( W_L \) and \( L \), taking into consideration 5, 6 and 7 we get, with some manipulations, the optimal conditions for wages and employment (see Appendix I):

\[
(W_L - \bar{W}_L) = \frac{1 - v}{v} \left[ W_L - \beta L_n^{-1} [(1 - n) l L_n]^{\beta} P_n(t) . A \left[ \int_0^J (q^{k(j,t)} x_n (k, j, t))^{\alpha} dj \right] \right]
\]

\( W_L = \left\{ \beta + \frac{\theta v (1 - \alpha - \beta)}{(1 - \theta) + \theta v} \right\} L_n^{-1} Y_L \)  \hspace{1cm} (14)

Equation (14) corresponds to the contract curve in the \((W_L, L)\) space. In other words, this corresponds to the relationship between wages and employment level that both firms and union agreed. On the other hand, equation (15) is the rent division curve or the low-skilled bargaining wage. As we have already seen, \( \beta L_n^{-1} Y_L \) as the marginal productivity of low-skilled labour.

**Result 2:** This contract curve is upward sloping in the \((W_L, L)\) plan if \( v > \frac{1}{2} \).

**Proof.** \( \frac{dW}{dL} = \frac{1 - v}{2v - 1} (1 - \beta) \beta [(1 - n) l] L_n^{-2} P_n(t) . A \left[ \int_0^J (q^{k(j,t)} x_n (k, j, t))^{\alpha} dj \right] \geq 0 \) iff \( 2v - 1 \geq 0 \).

\( \Box \)

**Result 3:** Given a particular level of employment, as the union’s bargaining power \( \theta \) increases, the negotiated wage rate will rise.

**Proof.** \( \frac{\partial (W_L - \bar{W}_L)}{\partial \theta} = \frac{v (1 - \alpha - \beta) [(1 - \theta + \theta v) + (1 - v) \theta]}{(1 - \theta + \theta v)} > 0 \)

\( \Box \)
**Result 4:** In the most general case, $0 < \theta < 1$, $0 < v < 1$, and $\alpha + \beta < 1$, $W^B_L > W^PC_L$.

**Proof.** $W^PC_L = \beta L^{-1}_n Y_L$. Since $\frac{\theta v (1-\alpha - \beta)}{(1-\theta + \theta v)} > 0$, unions have a positive impact on low-skilled wages. Generally, this can be seen as:

$$W^B_L = \frac{\beta L^{-1}_n M}{W^PC_L} + \frac{\theta v (1-\alpha - \beta)}{(1-\theta + \theta v)} L^{-1}_n M$$

positive impact of unions

In other words, the low-skilled wage will be higher that its marginal productivity.

These results are in line with Chang et al. (2007) and Lingens (2004). Figure 4 illustrates the bargaining case with the two possible situations: a upward sloping contract curve (Figure 4a) and a downward sloping contract curve (Figure 4b). Notice that the contract curve replaces the aggregate supply curve.

Combining 5, 6, 7, 9 and 15, we get the profit function of the representative firm:

$$\Pi_n = (1 - \alpha - \beta) \left[ \frac{(1 - \theta)}{1 - \theta + \theta v} P_n Y_L + P_n Y_H \right] \geq 0$$

(16)

where $P_n$ is the price of final good $n$.

Finally, combining 6 and 7 with 5, we get:

$$Y_n(t) = A^{1-\alpha} \left( \frac{\alpha P_n(t)}{P(j_t)} \right)^{\frac{\alpha}{\beta}} \left\{ \left[ (1-n) . L_n \right]^{\frac{\alpha}{\beta}} Q_L(t) + \left[ (n H_n)^{\frac{\beta}{\alpha}} \right] Q_H(t) \right\}$$

(17)

where: $Q_L = \int_0^J (q^L(j,t))^{\frac{\alpha}{\beta}} dj$ and $Q_H = \int_0^J (q^H(j,t))^{\frac{\alpha}{\beta}} dj$ are “two aggregate quality indexes, measuring the technological knowledge in each range of intermediate goods,
adjusted by market power that is the same for all the monopoly producers” (Afonso, 2006, p. 14). Normalizing the price of the composite final good at each time to one, we have:

\[ Y(t) = \int_0^1 P_n(t) Y_n(t) \, dn = \exp \left[ \int_0^1 \ln Y_n(t) \, dn \right] \quad (18) \]

The resources available in the economy, \( Y_n(t) \), can be used in R&D sector, \( R \), in the production of intermediate goods, \( X \), or consumed, \( C \).

\[ Y(t) = X(t) + R(t) + C(t) \]

2.3 Intermediate Sector

In order to produce, firms in the intermediate sector use one unit of aggregate output to obtain one unit of \( j \). Since \( P_n(t) = 1 \), this implies that the marginal cost of production is also one. Each quality of \( j \) is produced in a monopolist environment and the firm that holds the patent for the highest quality at \( t \) gets:

\[ \Pi_j(t) = (P(t) - 1) X_j(t) \quad (19) \]

Knowing that \( X_j = \int_0^1 X_{jn} \, dn \), and taking into consideration [6] and [7] we can maximize [19] in respect to \( P(k,j,t) \) to obtain the profit-maximization price of the monopolistic intermediate good:

\[ P(k,j,t) = \frac{1}{\alpha} \quad (20) \]

Notice that this monopoly price is: (i) a mark-up over the marginal cost of production, since \( 0 < \alpha < 1 \); (ii) constant across intermediate goods and time invariant; and (iii) independent of the quality level of the intermediate good. Following [Grossman and Helpman (1991), Ch. 4], we assume that limit pricing strategy is binding and it is used by all firms, i.e. \( p = q^\alpha \).

2.4 R&D sector

We follow the methodology developed by [Afonso and Leite (2010) and Afonso (2006)]. Briefly, investing in a new patent and, therefore, on a new quality level, closely depends on the profit-yields accrued by the monopolist and on the monopoly duration. Let \( pb(k,j,t) \) denote the probability of successful innovation in the next quality intermediate good \( j \) at time \( t \), which is given by:

---

8 We have assumed that all the intermediate goods that were bought by the final good firms fully depreciate at the end of each \( t \). For further information see [Acemoglu and Zilibotti (2001)].

9 For further details please see [Afonso and Leite (2010) and Afonso (2006)].
\[ pb(k,j,t) = rs(k,j,t) \cdot gq^{k(j,t)} \cdot \zeta^{-1} \cdot q^{-(1-\alpha)^{-1}k(j,t)} \cdot m^{-\frac{\alpha}{1-\alpha}} \cdot f(j) \] (21)

where:

(i) \( rs(k,j,t) \) corresponds to the total amount of aggregate final-good resources devoted to R&D in numeraire;

(ii) \( gq^{k(j,t)} \) is a learning-by-doing effect relating past successful R&D in \( j \) with the current probability of success (Grossman and Helpman (1991), Ch. 12; and Connolly (2003));

(iii) \( q^{-(1-\alpha)^{-1}k(j,t)} \) is an adverse effect, i.e., a cost of complexity (Kortum (1997));

(iv) \( m^{-\frac{\alpha}{1-\alpha}} \) corresponds to the adverse effect of market size - the bigger the size of the market measured by the labour employed, more difficult is to introduce new quality-adjusted intermediate goods and replacing old ones (Becker and Murphy (1992); Dinopoulos and Thompson (1999));

(v) \( f(j) \) apprehends the absolute advantage of high-skilled labour over low-skilled labour work with advanced technology (Nelson and Phelps (1966); Schultz (1975); Galor and Moav (2000)), where:

\[
f(j) = \begin{cases} 
1 & 0 \leq j \leq J; \ m = L \\
(1 + \frac{H}{H+L})^{\sigma} & J \leq j \leq 1; \ m = H \text{ and } \sigma = 1 + \frac{H}{L}
\end{cases}
\]

Finally, notice that (ii) and (iii) are modeled in order to offset the positive influence of the quality rung on the profits of each intermediate good leader (Afonso (2006)).

3 Equilibrium

We analyse the equilibrium of the model in three steps. First, we derive the equilibrium for a given technological-knowledge level. Second, we introduce the R&D activities and derive the aggregate spending in R&D as well as the law of motion of technological knowledge. Finally, we describe the transitional dynamics and the steady-state growth.

3.1 Equilibrium given a technological-knowledge level

The economic viability of either type of technology depends on: (i) the relative productivity, \( h/l \), (ii) the price of the \( m \)-type labour, (iii) the relative productivity and prices of the intermediate goods. The latter depends on complementarity with either \( m \)-type on the technological knowledge embodied and on the mark-up, both of these are summarised in the quality indexes \( Q_L \) and \( Q_H \).
Thus, from modeling the final-goods sector section, we can obtain the demand wages in both cases. Table 1 summarises the main results, combining 8, 9, 10, 15 and 17.

<table>
<thead>
<tr>
<th>Demand Wage</th>
<th>Perfect Competition</th>
<th>Monopoly Union</th>
<th>Nash-Bargaining</th>
<th>Either Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-skilled wage</td>
<td>$\beta \left[ L^{-1} A \psi \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} \right]^{-\gamma} \left[ \ln L \ln L \right]^{-\gamma} \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} P_{L}^{\frac{1}{\gamma}} Q_{L}(t)$</td>
<td>$\left[ 1 - (1 - n) \psi \right] \beta \left[ L^{-1} A \psi \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} \right]^{-\gamma} \left[ \ln L \ln L \right]^{-\gamma} \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} P_{L}^{\frac{1}{\gamma}} Q_{L}(t)$</td>
<td>$\left[ \beta + \frac{\theta(1 - \alpha - \beta)}{(1 - \beta + \theta v)} \right] \left[ L^{-1} A \psi \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} \right]^{-\gamma} \left[ \ln L \ln L \right]^{-\gamma} \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} P_{L}^{\frac{1}{\gamma}} Q_{L}(t)$</td>
<td>$\beta \left[ H^{-1} A \psi \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} \right]^{-\gamma} \left[ \ln H \ln H \right]^{-\gamma} \left( \frac{\alpha}{\gamma} \right) \frac{1}{\gamma n} P_{H}^{\frac{1}{\gamma}} Q_{H}(t)$</td>
</tr>
</tbody>
</table>

Table 1: Summary of the demand wages

Interestingly, notice that $A, Q_{L}, Q_{H}, L, H, h$ and $l$ are all independent of $n$. Therefore, it must be the case that in equilibrium $(1 - n) \frac{\alpha}{\gamma} \frac{1}{\gamma n} P_{L}^{\frac{1}{\gamma}}$ is constant across $n \in [0, \pi]$, and $n \frac{\alpha}{\gamma} P_{H}^{\frac{1}{\gamma}}$ is constant across $n \in [\pi, 1]$. (Afonso (2006)). Thus, combining 17 with these two prices expressions and knowing that, from the consumer’s maximization problem, $P_{n}(t) Y_{n}(t)$ is equal across all $n$, we can find an endogenous threshold final good $\bar{n}$, where $L$-technology is used in final goods $n \leq \bar{n}$ and $H$-technology in the final goods $n > \bar{n}$ (see Appendix II):

$$\bar{n} = \left\{ 1 + \frac{h H}{l L} \left( \frac{Q_{H}(t)}{Q_{L}(t)} \right)^{\frac{1}{1 - \alpha}} \right\}^{-1}$$

(22)

Moreover, taking into consideration that in the production on $\bar{n}$ both a firm that uses $L$-technology and a firm that uses $H$-technology should break even (Afonso and Leite (2010)), this threshold can be rewritten in terms of price indexes:

$$\frac{P_{H}(t)}{P_{L}(t)} = \left[ \frac{\bar{n}(t)}{1 - \bar{n}(t)} \right]^{\beta}$$

(23)

where:

$$f(j) = \begin{cases} P_{L} = P_{n} (1 - n)^{\beta} = \exp(-\beta) \bar{n}^{-\beta} & \text{if } j = L \\ P_{H} = P_{n} (n)^{\beta} = \exp(-\beta) (1 - \bar{n})^{-\beta} & \text{if } j = H \end{cases}$$

(24)

since $(\exp) \int_{0}^{1} \ln P_{n} dn = 1$.

From 22 we know the labour market structure will affect this threshold, which in turn will affect the direction of R&D activities through the price channel (e.g. Acemoglu (2002a)). Indeed, if the relative demand for workers, $\frac{H}{L}$, increases, this will affect the relative demand for technology, $\frac{Q_{H}}{Q_{L}}$, which, ceteris paribus, will also increase. This relationship is analysed in detail in the following section.
The next step is to find the equilibrium values for the macroeconomics aggregates. Combining 6, 7, 17, 22 and 24 we obtain (see Appendix III):

\[ X(t) = \int_0^1 \int_0^1 x_n(k, j, t) \, dk \, dn = \]

\[ \exp \left( -\beta \frac{1}{1-\alpha} \right) \left[ \frac{\alpha A}{1/\alpha} \right]^{\frac{1}{1-\alpha}} \left\{ \left( \left[ l L_n \right]^{\frac{\alpha}{1-\alpha}} Q_L \right)^{\frac{1}{2}} + \left( \left[ h H_n \right]^{\frac{\alpha}{1-\alpha}} Q_H \right)^{\frac{1}{2}} \right\}^2 \]  

(25)

\[ Y = \int_0^1 P_n(t) Y_n(t) \, dn = \]

\[ \exp \left( -\beta \frac{1}{1-\alpha} \right) A^{\frac{1}{1-\alpha}} \left[ \frac{\alpha}{1/\alpha} \right]^{\frac{1}{1-\alpha}} \left\{ \left( \left[ l L_n \right]^{\frac{\alpha}{1-\alpha}} Q_L \right)^{\frac{1}{2}} + \left( \left[ h H_n \right]^{\frac{\alpha}{1-\alpha}} Q_H \right)^{\frac{1}{2}} \right\}^2 \]  

(26)

Finally, the equilibrium in the labour market will depend on the bargaining structure.

3.1.1 Perfect competition

Combining 22 and 23 with the perfect competition demand wages in Table 1, we obtain the relative demand:

\[ \omega^D = \left( \frac{W_H}{W_L} \right)^D = \left[ \left( \frac{H^D}{L^D} \right)^\Psi \left( \frac{h}{l} \right)^{\frac{\alpha}{1-\alpha}} \frac{Q_H}{Q_L} \right]^\frac{1}{2} \]  

(27)

where \( \Psi = \frac{\beta - 2(1-\alpha)}{(1-\alpha)} \). Since, in equilibrium, \( \omega^S = \omega^D \), where \( \omega^S = \frac{H^S}{L^S} \) from 3,

\[ \left[ \left( \frac{H}{L} \right)^{PC} \right]^* = \left[ \left( \frac{h}{l} \right)^{\frac{\alpha}{1-\alpha}} \frac{Q_H}{Q_L} \right]^\Theta \]  

(28)

where: \( \Theta = \frac{1}{2(\gamma - 1) - \Psi} \)

Finally, we get:

\[ \left[ \omega^{PC} \right]^* = \left\{ \left( \frac{h}{l} \right)^{\frac{\alpha}{1-\alpha}} \frac{Q_H}{Q_L} \right\}^{\gamma - 1} \]  

(29)

where \( \omega^{PC} \) corresponds to the wage ratio under perfect competition. Equations 28 and 29 correspond to the perfect equilibrium solution and they will be used as a baseline case.

3.1.2 Monopoly case

In what relates with the monopoly situation, from section 2.2.3, we know that:
\[ W_L^M = z.W_L^{PC} \]

where \( z = \frac{1-(1-\nu)\beta}{\nu} \) and \( W_L^M \) corresponds to the monopoly low-skilled wage. Defining \( w_L^M \) as the wage ratio under monopoly situation, we get:

\[ [\omega^M]^* = \frac{W_H^{PC}}{W_L^M} = \frac{W_H^{PC}}{z.W_L^{PC}} = 1 - \frac{1}{z} \left\{ \left( \frac{h}{l} \right)^{\frac{\alpha}{\beta}} \frac{Q_H}{Q_L} \right\}^\theta \gamma^{-1} \] \hspace{1cm} (30)

**Result 5:** Ceteris paribus, \([\omega^M]^* < [\omega^{PC}]^*\)

**Proof:** Since \( z > 1 \), it follows straightforward from (30).

In this case, the level of employment of low-skilled workers would be determined on the demand curve, as Figure 4 illustrated.

\[ \frac{1}{z} \left\{ \left( \frac{h}{l} \right)^{\frac{\alpha}{\beta}} \frac{Q_H}{Q_L} \right\}^\theta \gamma^{-1} = \left( \frac{H}{L} \right)^{\Psi} \left( \frac{h}{l} \right)^{\frac{\alpha}{\beta}} \frac{Q_H}{Q_L} \]

\[ \left( \frac{H}{L} \right)^M = \frac{1}{z} \left( \frac{h}{l} \right)^{\frac{\alpha}{\beta}} \frac{Q_H}{Q_L} \] \hspace{1cm} (31)

**Result 6:** Ceteris paribus, \([\left( \frac{H}{L} \right)^M]^* > [\left( \frac{H}{L} \right)^{PC}]^*\)

**Proof.** Since \( z > 1 \) and \( \frac{2}{\Psi} = \frac{2(1-\alpha)}{\beta(1-\alpha)} = \frac{2(1-\alpha)}{\beta-2(1-\alpha)} < 0 \), because \( \beta - 2(1-\alpha) < 0 \), \([\left( \frac{H}{L} \right)^M]^* > [\left( \frac{H}{L} \right)^{PC}]^*\)

3.1.3 Bargaining case

Regarding the bargaining situation, it is possible to prove that the low-skilled wage will also be a mark-up over the perfect competition wage, as follows:

\[ W_L^B = K.W_L^{PC} \]

where: \( K = \left( 1 + \frac{\theta(1-\alpha-\beta)}{\beta(1-\theta)+\theta} \right) \). Therefore, defining \( \omega^B \) as the wage ratio under the bargaining case, we have:
\[ \left[ \omega^B \right]^* = \frac{W_{H}^{PC}}{W_{L}^{B}} = \frac{W_{H}^{PC}}{K W_{L}^{PC}} = 1 - \frac{1}{K} \left\{ \gamma \right\} \left( \frac{h}{T} \right)^{\frac{\beta}{\gamma}} \frac{Q_{H}}{Q_{L}} \right]^{\theta} \]  \quad (32)

**Result 7:** Ceteris paribus, \( \left[ \omega^B \right]^* < \left[ \omega^{PC} \right]^* \)

**Proof:** Since \( K = \left( 1 + \frac{1}{\beta} \frac{\theta_{v} \left(1 - \alpha - \beta \right)}{(1-\theta + \theta_{v})} \right) = \frac{\beta \left(1 - \theta + \theta_{v} \right)}{\beta \left(1-\theta + \theta_{v} \right)} > 1 \), it follows straightforward from (32).

In this case, however, the level of employment will be determined on the contract curve. Combining (14), (27) and (29) we can define the relative contract curve and the equilibrium relative demand for the bargaining case (Appendix IV):

\[ \left( \frac{W_{H}}{W_{L}} \right)^{CC} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} \left( \frac{H^{D}_H}{L^{C}} \right)^{\psi} \left( \frac{h}{T} \right)^{\frac{\beta}{\gamma}} \frac{Q_{H}}{Q_{L}} \right]^{\theta} \]  \quad (33)

\[ \left[ \left( \frac{H}{L} \right)^{B} \right]^* = B^{\psi} \left[ \left( \frac{h}{T} \right)^{\frac{\beta}{\gamma}} \frac{Q_{H}}{Q_{L}} \right]^{\theta} \]  \quad (34)

**Result 8:** Ceteris paribus, \( B \gg 1 \) iff \( v \ll \frac{1}{2} \).

**Proof.**

\[ \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} \]  \quad (32)

\[ \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} + \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} = \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} \]. In this case, since \( \frac{\theta_{v} \left(1 - \alpha - \beta \right)}{(1-\theta + \theta_{v})} > 0 \), we just have to take into consideration \( 1 - \frac{1}{v} \). Thus, if \( v \ll \frac{1}{2} \), \( B \gg 1 \).

\[ \frac{1 - \frac{1}{v}}{1 - \frac{1}{v}} \]

**Result 9:** Ceteris paribus, \( \left( \frac{H}{L} \right)^{B} \geq \left( \frac{H}{L} \right)^{PC} \) iff \( B \leq 1 \).

**Proof.** Since \( \frac{2}{\psi} = 0 \), \( B > 1 \) implies that \( \left[ \left( \frac{H}{L} \right)^{B} \right]^* < \left[ \left( \frac{H}{L} \right)^{PC} \right]^* \) and otherwise if \( B < 1 \).

Therefore, the results within the bargaining case are not as straightforward as in the monopoly situation. Indeed, the impacts on wages and employment will depend on the union’s preferences. Although, ceteris paribus, \( \omega^B < \omega^{PC} \); \( \left( \frac{H}{T} \right)^{B} \geq \left( \frac{H}{T} \right)^{PC} \) if \( v \ll \frac{1}{2} \) (and thus \( B \ll 1 \)). This result is in line with Result 2.
3.1.4 Growth path wages

However, in the three situations, perfect competition, monopoly union or bargaining, the growth path of low-skilled and high-skilled wages are the same:

\[
\hat{W}_L = \frac{1}{1 - \alpha} \hat{P}_L + \hat{Q}_L
\]

\[
\hat{W}_H = \frac{1}{1 - \alpha} \hat{P}_H + \hat{Q}_H
\]

3.2 Equilibrium with R&D

To determine the aggregate spending in R&D, we must understand how R&D is carried out in the intermediate-goods sector. We need to (i) determine which firms conduct R&D activities; (ii) determine the value of an innovation; and (iii) derive the laws of motion of \( Q_L(t) \) and \( Q_H(t) \).

Suppose the innovation is introduced by an outsider firm. From the profit maximization in the intermediate sector, we know that changes in profits are given by:

\[
\Delta \Pi_j = \Pi_j(\tau) - 0 = (P_j(\tau) - 1) X_j = (P_j(\tau) - 1) \left( \frac{P_m A \alpha}{P_j} \right)^{\frac{1}{1 - \alpha}} (\bar{m} m)^{\frac{\alpha}{1 - \alpha}} \left[ q^k_j(\tau) \right]^{\frac{\alpha}{1 - \alpha}}
\]

where: \( \bar{m} = h \) for \( m = H \) and \( \bar{m} = l \) for \( m = L \).

Following Afonso and Leite (2010), Barro and Martin (2004) and Aghion and Howitt (1992), it can be shown that it is more profitable to introduce a new quality of \( j \) by an outside firm than by the current monopolist.\(^{10}\) Moreover, it is also possible to prove that \( E[V(k, j, t)] \), “the expected current value of the flow of profits to the monopolist producer of intermediate good \( j \) (...) relies on the profits at each time, \( \Pi(k, j, t) \), on the given equilibrium interest rate and on the expected duration of the flow” (Afonso (2006), p. 17). Thus, we get:

\[
E[V(k, j, t)] = \frac{\Pi(k, j, t)}{r(t) + p_b(k, j, t)}
\]

Regarding R&D activities, two main assumptions are needed: (i) free entry in R&D activities of the intermediate-goods sector; (ii) when an innovation is introduced as a consequence of R&D efforts of many firms, the probability of a firm becoming the successful innovator is proportional to its share on aggregate R&D. Implicitly, the R&D spending

\(^{10}\)Notice that in the scenario in which the latest innovation in intermediate good \( j \) is introduced by the current monopolist in that good, we have: \( \Delta \Pi_j^m = \Pi_j(\tau) - \Pi_j(\tau - 1) \) implying therefore that \( \Delta \Pi_j^m(\tau) = \left( \left( \frac{1}{\alpha} \right)^2 - 1 \right) \left( \frac{P_m A \alpha}{(Y_m)^{\alpha}} \right)^{\frac{\alpha}{1 - \alpha}} (\bar{m} m)^{\frac{\alpha}{1 - \alpha}} \left[ q^k_j(\tau) \right]^{\frac{\alpha}{1 - \alpha}} \).
in order to improve \( j \) should be equal to the expected payoff generated by the innovation, \( i.e. \) \( rs(k-1,j,t) = pb(k-1,j,t) V(k,j,t) \). In order words, we have:

\[
rs(k,j,t) = pb(k,j,t) V(k+1,j,t) \tag{37}
\]

Given the equilibrium aggregate R&D spending, \( R(t) = \int_0^1 rs(k,j,t) \, dj = \int_0^1 rs(k,j,t) \, dj + \int_1^j rs(k,j,t) \, dj \), and combining it with \( 21, 36 \) and \( 37 \) we obtain (see Appendix V):

\[
R = \int_0^1 rs(k,j,t) \, dj = \zeta \left[ Q_L L pb_L + Q_H H pb_H \right] \tag{38}
\]

where:

\[
 pb_L = \frac{\theta}{\zeta} f(j) t^{\alpha} \left( \frac{q-1}{q} \right) \left[ P_L(t) A^\alpha \right]^{\frac{1}{1-\alpha}} - r(t)
\]

\[
 pb_H = \frac{\theta}{\zeta} f(j) h^{\alpha} \left( \frac{q-1}{q} \right) \left[ P_H(t) A^\alpha \right]^{\frac{1}{1-\alpha}} - r(t)
\]

are the probabilities of successful R&D, which are independent of \( j \) and \( k \).

Finally, regarding the law of motion of \( Q_m(t) \), suppose that a new quality of intermediate good \( j \) is introduced. All else remaining equal, the change in the corresponding aggregate quality indexes is given by:

\[
\Delta Q_m = (q^{k+j+1})^{\frac{1}{1-\alpha}} - (q^{k-j})^{\frac{1}{1-\alpha}} = (q^k)^{\frac{1}{1-\alpha}} \left[ q^{\frac{\alpha}{1-\alpha}} - 1 \right] \tag{39}
\]

Thus, combining \( 36, 37 \) and \( 38 \) the following equation arises (see Appendix VI):

\[
\dot{Q}_m(t) = \left[ \frac{\theta}{\zeta} f(j) \bar{m} t^{\alpha} \left( \frac{q-1}{q} \right) \left[ P_m(t) A^\alpha \right]^{\frac{1}{1-\alpha}} - r(t) \right] \left[ q^{\frac{\alpha}{1-\alpha}} - 1 \right] \tag{40}
\]

### 3.3 Transition dynamics and steady-state growth

Taking into consideration \( 25, 26, 38 \) and that \( Y = X + R + C \), all macroeconomic aggregates, \( Y, X, R \) and \( C \) can be expressed as multiples of the aggregate quality index, \( Q_L(t) \) and \( Q_H(t) \). This implies that the path of all relevant variables outside the steady-state, including that of the wage ratio, depend on a single differential equation that governs the paths of the technological-knowledge bias, \( i.e., \) the path of \( \dot{D}(t) = \frac{D(t)}{D(t)} = \frac{Q_H(t)}{Q_H(t)} - \frac{Q_L(t)}{Q_L(t)} \). Therefore, combining \( 24, 38 \) and \( 40 \) we obtain:

\[
\dot{D} = \left[ \frac{\theta}{\zeta} \left( \frac{q-1}{q} \right) \left[ A^\alpha \right]^{\frac{1}{1-\alpha}} \left[ q^{\frac{\alpha}{1-\alpha}} - 1 \right] \exp \left( \frac{-\beta}{1-\alpha} \right) \times
\]

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\[
\left( h^{\frac{\beta}{\bar{\alpha}}} \left( 1 + \frac{H}{H + L} \right)^{\sigma} \left( 1 + \left[ D(t) \frac{h H}{L} \right]^{-\frac{1}{2}} \right)^{\frac{\beta}{\bar{\alpha}}} - \left( 1 + \left[ D(t) \frac{h H}{L} \right]^{\frac{1}{2}} \right)^{\frac{\beta}{\bar{\alpha}}} \right) \right) \frac{d}{dt} \hat{\sigma} \]  

(41)

Finally, along the balanced growth path all variables grow at the same constant rate. Recalling that \( \hat{C} = \frac{r(t) - \rho(t)}{\phi} \), we then know that:

\[ g^* = \hat{Q}_H^* = \hat{Q}_L^* \]  

(42)

4 Sensitivity Analysis

4.1 Baseline Analysis

In this section we aim to analyse and compare the three situations in the labour market structure regarding: (a) technological-knowledge bias; (b) wage premium; (c) employment; (d) unemployment. Since \( \frac{H}{L} \) crucially depends on the labour market situation, the direction of technological-knowledge progress and its repercussion on wage premium are expected to be different among the three possible cases.

Using the fourth-order Runge-Kutta classical numerical method, we can analyse the behaviour of the technological-knowledge bias for a set of baseline parameter values, as follows: \( A = 1.50, h^a = 1.20, l^a = 1, \alpha^b = 0.7, \beta^b = 0.2, q^b = 3.33, q^a = 1.60, \zeta^a = 4.00, \phi^e = 1.50, \rho^d = 0.02, \) and \( \gamma = 3.10 \). \(^{[11]}\)

4.1.1 Perfect Competition

By replacing \( \left( \frac{H}{L} \right)^{PC} \) into \( \hat{D}(t) \), we can study the equilibrium path and its dynamics under perfect competition. Figure 5 summarizes the main results regarding the transition dynamics. As expected, there is technological-knowledge bias towards the high-skilled technology which therefore implies a wage ratio greater than one. These results are in line with Afonso (2006).

In what relates with the labour market dynamics, Figure 6 illustrates the results. Since we have perfect competition, \( \left( \frac{H}{L} \right)^D = \left( \frac{H}{L} \right)^S \), there is no unemployment. This relationship will be crucial in the next two sub-sections.

\(^{[11]}\) Notes:

a) The values are in accordance with the theoretical assumptions, such that: (i) \( h > l \geq 1 \) – see 3; (ii) \( g > 0 \) – see Equation 21; (iii) \( \zeta > 0 \) – see Equation 21; (iv) \( \gamma > 1 \) – see Equation 1.

b) These values are in line with the mark-up estimates in, e.g., Kwan and Lai (2003).


d) Set in line with previous works on growth (see, e.g., Dinopoulos and Thompson (1999)). Source: Author’s assumptions, based on theoretical framework and on the literature.
4.1.2 Monopoly case

Applying the same logic, we replace \( \left( \frac{H}{L} \right)^M \) into \( \tilde{D}(t) \) in order to study the equilibrium path and its dynamics under a monopoly union. Figure 7 provides a graphical representation of the transition dynamics and Figure 8 its relationship with labour market, where \( v = 0.8 \).

From \( t = [1, 10] \) the economic is under perfect competition in the labour market. At \( t = 10 \) a monopoly union is introduced, which immediately affects the steady-state paths of all four variables. Starting the analysis with \( H \)-premium, an immediate drop is verified, as one might expected - see Figure 3. Indeed, the monopoly union will demand a higher low-skilled wage when compared with the perfect competition scenario, which fully account for this fall in wage ratio and also explains the immediate rise (fall) in relative demand (supply) - see Result 5 and Result 6.

Regarding the technological-knowledge bias, due to the rise in the relative demand,
we observe an increase in the technological-knowledge-absorption effect - in Equation 21, \( f(j) \) jumps immediately from 3.309 to 4.797 as a result of the introduction of a monopoly union.

However, from period 10 onwards, we observe an increase in all four variables, with especial attention for the technological-knowledge bias and the wage ratio, which can be intuitively explained as follows: facing a rise in the low-skilled wages, firms start increasing (decreasing) their demand for high(low)-skilled workers, which in turn leads to an increase(decrease) in the demand for high(low)-skilled technology. On the other hand, this increase in the technological-knowledge bias leads to a decrease in the demanded monopoly wage - notice that \( W^M \) depends on \( \frac{Q_H}{Q_L} \). Indeed, there is a backfire effect: the union fails to anticipate this variation in \( \frac{Q_H}{Q_L} \) and it does not have any option rather than decrease the monopoly wage. Thus, the relative demand shifts to the right and the relative monopoly wage upwards. Figure 8 provides a graphical representation of these dynamics.

Moreover, from Afonso (2006), we already know that changes in \( H \)-premium are closely related with changes in technological-knowledge bias - see Equation 30. Indeed, an increase in \( \frac{Q_H}{Q_L} \) leads to an increase in the supply of \( H \)-intermediate goods, which therefore increases the number of final goods produced with \( H \)-technology and lowers their relative price - see Equation 22 and Equation 23. Therefore, relative prices of final goods produced with \( H \)-technology fall continuously towards the new constant steady-state levels, implying that \( \frac{Q_H}{Q_L} \) is increasing, but at a decreasing rate until it reaches its new higher steady-state value.

However, in this case, since labour market dynamics are endogenous to the model, this change in relative demand is not only an endogenous mechanism to the model (in
contrast with Afonso (2006), but it also enhances the relationship between $H/L$ and $Q_H/Q_L$ due to the shift of relative demand to the right.

The new equilibrium is characterised by a higher low-skilled wage as well as a higher relative wage, but also by the existence of unemployment. Since the high-skilled wage can fully adjust, this unemployment corresponds to low-skilled unemployment due to an increase of its wage (from $L^S$ to $L^D$, Figure 8a). Finally, it is interesting to note that within this framework, unions actually contribute to the increase of wage dispersion - although they can increase the low-skilled wage, the increase in relative demand leads to a higher relative wage in the new steady-state.

![Diagram](image)

(a) Labour market dynamics

![Diagram](image)

(b) Wage and employment ratio

Figure 8: Monopoly union

4.1.3 Bargaining case

In this last situation, we replace $\left[\left(\frac{H}{L}\right)^B\right]^*$ into $\hat{D}(t)$. However, we have to taking into consideration the value of $v$, the union’s preferences for wage or employment. Indeed, two cases arise.

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Case 1: $\theta = 0.5$ and $v > 0.5$

Figure 10 describes this situation. From $t = [0, 10]$, the economy is again under perfect competition in the labour market. At $t = 10$, firms and union start negotiating low-skilled wages and low-skilled employment, which immediately affects the steady-state paths of all four variables. Regarding $H$-premium, an immediate drop is verified, as one might expected - see Figure 4a). Indeed, notice that the low-skilled wage is now determined under the contract curve, which breaks down the standard negative relationship between demanded wages and employment. This implies that, although the negotiated low-skilled wage is higher than the wage under perfect competition, meaning a lower relative wage, we observe a decrease in the relative demand - see Result 7 and Result 8. This is in contrast with the monopoly case and it is what explains the immediate fall in the technological-knowledge-absorption effect, from 3.309 to 2.833.

Moreover, from period 10 onwards, a decrease in all four variables is verified, which is again in contrast with the previous situation, and can be explained as follows: under this type of efficient bargaining, firms agreed to employ more low-skilled workers when compared with the perfect competition scenario, which, due to the existent complementarities between inputs, implies that it is now more profitable to invest in low-skilled technology, since $H_L$ is now lower - see Equation 4. Notice that this is the same mechanism present in Afonso and Leite (2010) and Afonso (2006), which is, once again, enhanced due to the shift of the relative demand to the left - see Figure 10. Regarding the relative wage, since the contract curve is upward slopping, a decrease in $H_L$ is follows by a decrease in $W_L/W_H$.

The new equilibrium is characterised by: (i) higher low-skilled wage but (ii) a lower relative wage, implying therefore that within this framework unions can actually contribute
for lowering the wage dispersion. Finally, it is interesting to note that the contract curve is at the right of the supply curve (Figure 10a), which means that in this case there is no unemployment - for any wage above the perfect competition scenario, all workers who aim to work are, indeed, employed.

(a) Labour market dynamics

(b) Wage and employment ratio

Figure 10: Bargaining case, $\theta = 0.5$ and $\nu = 0.6$

**Case 2: $\theta = 0.5$ and $\nu < 0.5$**

Figure 11 presents this situation. Once again, firms and unions start bargaining $W_L$ and $L$ at $t = 10$, which affects the paths of all four variables instantly. Regarding the wage ratio, an immediate drop is once more verified - see Figure 4b). As in the previous case, the low-skilled wage is determined under the contract curve, but since it is now downward sloping, the standard negative relationship between wages and employment is kept. This fully accounts for the jump in the relative demand, which is in contrast with the previous case but in line with the monopoly situation. Thus, a jump in $Q_H Q_L$ is also verified, due to the technological-knowledge-absorption effect, from 3.309 to 3.601.

From period 10 onwards, an increase in all four variables is observed, which contrasts with the previous bargaining case, but it is in line with the monopoly dynamics, with a small exception - although the wage ratio starts increasing after $t = 10$, in the new steady-state it will still be lower when compared with the perfect competition wage. This
is not the case with a monopolistic union, since \( (W_H/W_L)^M > (W_H/W_L)^{PC} > (W_H/W_L)^B \). Indeed, since the contract curve is on the right of the demand curve (see Figure 5b), for the same low-skilled wage, firms do have to employ more workers than their initial demand. This helps to explain why firms do not fully adjust their demand for low-skilled workers. In what regards the technological-knowledge bias, the adjustment in the relative demand is followed by an adjustment in the \( Q_H/Q_L \), which is in line with \( \text{Afonso (2006)} \). Once again, this effect is enhanced by the shift in the relative demand, but it is not enough to return wages at their initial level - see Figure 12. Notice that the backfire effect is also present in this case.

\[
\frac{Q_H}{Q_L} = 10.96; \frac{W_H}{W_L} = 2.273
\]

Figure 11: Bargaining case - \( \theta = 0.5 \) and \( v = 0.4 \)

Therefore, the new equilibrium is characterised by a higher low-skilled wage and a lower relative wage, which is in line with the results from the previous type of bargaining. However, since now the contract curve on the left of the supply curve, there is unemployment. Taking into account that, once more, the high-skilled wage can fully adjust, this unemployment corresponds to low-skilled unemployment (from \( L^S \) to \( L^D \), Figure 12a) due to an increase of its wage. Interestingly \[ \left( \frac{W_H}{W_L} \right)^M > \left( \frac{W_H}{W_L} \right)^B \] and \[ \left( \frac{H}{L} \right)^M > \left( \frac{H}{L} \right)^B \], which seems counter intuitive - to a high relative wage corresponds a higher relative demand. Nevertheless, notice that within the monopoly(bargaining) framework the new relative wage is above(below) the perfect competition wage, due to the fact that the relative demand reacts more(less) than proportionally to the introduction of a trade union.
4.2 Discussion

In this section we aim to perform a simple sensitivity analysis regarding the impacts of the two different bargaining structures on the technological-knowledge bias, relative wages, demand and supply. In particular, we analyse how unions’ preferences, $0 \leq \nu \leq 1$, and bargaining power, $0 \leq \theta \leq 1$, can influence these variables. Table 2 summarises the main results. It is interesting to note that: (i) the relationship between $H_L$ and $Q_H$, in which a higher employment ratio implies a higher technological-knowledge bias, is always present in all the considered cases. As we stated before, this behaviour is closely related with the existent complementaries between inputs - see Equation 4; (ii) a(n) lower(higher) $\theta$ leads the economy to a closer(more distant) equilibrium comparing with the perfect competition case; (iii) with $\nu = 0.5$, $(H_L)^D = (H_L)^S = (H_L)^{PC}$, meaning that the contract curve is vertical and the wage level will depend only on the bargaining power of each part. Nevertheless, it will be always higher than the perfect competition wage if $\theta > 0$.

Moreover, in order to understand the relative impact of the labour share on the technology-knowledge bias and wages, we perform a small change in the values of $\alpha$ and $\beta$, the share in production of the intermediate goods and labour, respectively, from $\alpha = 0.7$ to $\alpha = 0.6$ and $\beta = 0.2$ to $\beta = 0.3$. In other words, we are taking into consideration a new production structure where labour has a higher contribution when compared with
the first situation. Table 3 summarises the main results. It is interesting to note that: (a) $\frac{W_H}{W_L}$ is now higher in all four situation, with the exception of the monopoly case, which seems to be constant. This implies that in terms of wages high-skilled workers seem to benefit more from this change in production structure; (b) $\frac{Q_H}{Q_L}$ (and $H^D_L$) is higher only for the bargaining case with $v > 0.5$. Intuitively, since now labour plays a more important role in the production function, and in this case firms are tied to the positive relationship between low-skilled workers and low-skilled wages, they prefer to increase more than proportionally the high-skilled technology and, therefore, $H_L$; (c) taking into account that $(H_L)^S$ do not depend directly on $\beta$, there are only some very small changes in $[(H_L)^S]^*$; (d) regarding the unemployment level, since relative demand decreases proportionally more than relative supply, this change in the production structure lowers low-skilled unemployment.

Finally, the mix results that we obtained regarding the possible and different effects that trade unions might have in SBTC and employment are somehow in line with Figure 1 and with the mix results that one can find in the empirical works. From our analysis, it seems clear that in order to empirically analyse the impacts on trade unions one need to control not only for the institutional framework, namely the trade union density, the level of employment protection legislation, unemployment benefits or the net replacement ratio, which can in part account for the bargaining power of unions, but also for the bargaining
structure *per se*, since, as we have seen, its impacts differ from case to case.

5 Concluding Remarks

We analysed the relationship between skill-biased technological change and employment, under a collective bargaining structure perspective. We focused our analysis of the role on trade unions and their impact on skill-premium and employment. Our analysis suggests that: (i) introducing labour market imperfections in the final good sector will affect not only the wage ratio, but also the dynamics of the most important macroeconomic variables, namely the aggregate R&D spending, the aggregate quality indexes and employment; (ii) trade unions can actually increase low-skilled wages without increasing or even implying low-skilled unemployment, if they have some bargaining power and care more about employment than wages; (iii) two effects arise from the introduction of a trade union: the *backfire effect*, when the initial demanded low-skilled wage is higher than the steady-state wage, due to the fact that trade unions fail to predict their impacts on the technological-knowledge bias; and the *relative demand effect*, where firms adapt their demand for high- and low-skilled workers; (iv) a monopoly trade union fails to decrease the wage dispersion between high- and low-skilled workers as a result of the combination between the relative demand effect and the backfire effect; (v) the technological-knowledge bias follows the path of the relative demand, which will be affected by the introduction of trade unions; (vi) changes in the production structure have different impacts, depending on the bargaining structure. These results are important to endogenously explain (a) why an increase in high-skilled workers do not necessary lead to a decrease in wage-premium; (b) and account for the mix evidence regarding the relationship between trade union and wages.

As future research, we aim to (a) apply our methodology to the European case, namely through a “north-south” analysis; (b) introduce the possibility of education and training as an option to low-skilled workers; and (c) analyse the role of public R&D funding.

**Note:** All appendices are available upon request.

References


