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Spatial Competition in a Circular Market With Delivery Direction Choice[1]

Competição Especial Num Mercado Circular Com Livre Escolha de Direção

Yuan-Chang Cheng
peanutcyc@gmail.com
SunWay Biotech Co., LTD

Fu-Chuan Lai
uiuclai@gate.sinica.edu.tw
Research Center for Humanities and Social Sciences, Academia Sinica and
Department of Public Finance, National Chengchi University

Abstract/ Resumo

This paper employs two concepts which are endogenous direction choices on product delivery and the first-entrant-take-all rule to capture the major characteristics on some utility industries, such as the natural gas or electric industries. It is shown that there are two equilibrium outcomes in a circular market. One outcome is that the two firms are located back-to-back at one point and transport their goods in opposite directions. The second outcome is that both firms are located equidistantly from each other and deliver the products in the same direction. These results are striking in that agglomeration location is one of the equilibrium patterns.

Keywords: Spatial Competition; Circular market; Directional markets.

JEL codes: D43; L13

Este artigo usa dois conceitos que são o de escolhas direcionais endógenas na remessa de produtos e a regra do primeiro-a-entrar-ganha-tudo para capturar as principais características de alguns ramos de serviços de utilidade pública, como as indústrias do gás natural ou elétrica. É mostrado que existem dois resultados de equilíbrio num mercado circular. Um dos resultados é que as duas empresas estão localizadas lado a lado num ponto do espaço e enviam os seus produtos em direções opostas. O segundo resultado é que ambas as empresas estão localizadas de forma equidistante entre si e entregam os produtos na mesma direção. Esses resultados são relevantes já que a localização com aglomeração é um dos padrões de equilíbrio.

Palavras-chave: Competição espacial; Mercado circular; Mercados direcionais

Código JEL: D43; L13

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1. INTRODUCTION

In 1494, Spain and Portugal signed the Treaty of Tordesillas which divided the newly discovered lands outside Europe along a meridian about 1770 km west of the Cape Verde Island. According to the treaty, the lands to the east would belong to Portugal and the lands to the west would belong to Spain. After that, the Portuguese explored to the Far East Asia via Africa, while the Spanish arrived in the Philippines via America. In a global sense, they engaged a competition in different exploration directions and once they reached a new land, they would be the monopoly owner of that land. In a theoretical sense, this paper develops a spatial competition model with a circular market to analyze the duopoly firms' direction and location equilibrium.

In the early spatial competition model, Hotelling (1929) claims that two firms competing in prices will position themselves in a "back-to-back" configuration in a linear market. However, d'Aspremont et al. (1979) show that two firms will disperse at two endpoints of a linear market in the Hotelling model by considering the quadratic transport costs form. Traditional wisdom says that two sellers would not be agglomerated at one point in the Hotelling-type models because firms will suffer from intense price competition (à la Bertrand) and they may undercut their prices to capture the whole market.

In reality, drastic competition at the agglomeration point of a Hotelling-type model seems to conflict with the phenomenon. Anderson and Neven (1991) established a spatial Cournot competition model to explain the common observation of agglomeration. Meanwhile, they point out that spatial Cournot competition is frequently used in the energy market, such as in the oil or natural gas industries, but they do not further analyze the characteristics of these industries.

An obvious feature of the energy industry, for example in the natural gas companies, is that users only pay expenses to the businesses which first provide the service.[2] This is because each house generally has only one natural gas outlet. This might cause a firm to urgently reach the local market. Similar examples include some utility industries, such as the telephone, Internet, cable TV services, and the discovery competition between Spain and Portugal. All of these industries exhibited a phenomenon of a first entrant advantage. This paper will employ a "first-entrant-take-all" rule to capture this "one-house-one-outlet" factor for each kind of service.

Another important feature of these utility industries is that there are large fixed costs. One possible reason is a great economy of scale. For example, a gas company makes large investments in creating and maintaining the delivery pipes and hence these facilities involve vary large fixed costs. In contrast, providing extra units of gas only spends a very small marginal cost. Generally speaking, they lay pipelines in order to distribute fuel to their users subject to a minimization on delivery costs. To simplify the enormous investments on pipelines in our modeling,[3] it is assumed that firms devote themselves to building a unidirectional product delivery. For this reason, the scheme of delivery direction through pipelines must be first examined.[4] Therefore, it is

[2] Since the 1990s, many public utility industries started their deregulation by introducing competition via access charges paid to the "last-mile" owners (see Baumol 1983, Baumol and Sidak 1994, Laffont and Tirole 1994, and Armstrong et al. 1996). However, this deregulation tide is not our focus. Actually, deregulation does not happen in every country. For example, in Taiwan today every natural gas company is still a local monopolist.

[3] We may imagine that every pipeline needs a host machine and some fixed equipment, then a one-direction pipeline will be cheaper than two pipelines with the same total distance to supply in two directions.

[4] Cancian et al. (1995) first incorporated directional constraints into the traditional location theories. They considered the scheduling problem of news-broadcasting among several TV companies. They showed that if many (at least two) firms simultaneously choose which time to start their broadcasting, there is no pure strategy Nash equilibrium since each station will respond by moving its news to start just before its rival's. After this, Lai (2001) constructed a directional Hotelling model with a sequential entry. A fish-catching game was introduced where two players choose locations individually in order to maximize their catches in a linear waterway in which all the fish swim to the

important for a firm to decide on its site when they face a limitation on the choice of directions.[5]

Previous literature on directional markets (Cancian et al., 1995, Lai, 2001) are only limited to studying the linear market, except Sun (2010), where he shows that the duopoly firm will agglomerate at one point if they choose their locations and direction at the first stage and choose their quantities at the second stage. Note that there is no "first-entrant-take-all" rule in Sun (2010), while his rationale of unidirectional delivery assumption is that a firm which delivers all goods in a circular city with a single truck can therefore only deliver in one particular direction, otherwise two dispatches are required, which is more expensive.

In this paper, we incorporate the first-entrant-take-all (one-house-one-outlet) rule and the endogenous choice of delivery directions into a spatial Cournot competition on a circular market which was first introduced by Pal (1998) and followed by Shimizu (2002), Chamorro-Rivas (2000) and Yu and Lai (2003). In contrast to the agglomeration results in a linear market (Anderson and Neven, 1991), Pal (1998) show that two firms will be located equidistantly from each other in a circular market.[6] This result seems to imply that Cournot oligopolists will choose to agglomerate or disperse in terms of market configurations. However, Matsushima (2001) demonstrates a counter-example to Pal's (1998) results. Gupta et al. (2004) identify multiple location patterns in a circular market and show that both Pal's (1998) and Matsushima's (2001) results are only two of their special cases.

Note that a striking feature of this paper is the endogenizing of the choice of delivery directions in a circular market with a restriction of "first-entrant-take-all" (one-house-one-outlet). It shows that there are two equilibrium outcomes. One is that two firms cluster at one point and choose to transport the goods in opposite directions, and the other is that both are located equidistantly on a circle and decide to deliver the products in the same direction.

The rest of this paper is organized as follows. In Section 2, the model is described. Section 3 solves and analyzes the equilibrium outcomes of the model. Finally, our conclusions are offered in Section 4.

2. MODEL

Suppose that there are two firms $\{1,2\}$, which are identical in every aspect engaging in a potential Cournot competition on a circular market with a perimeter equal to 1. Consumers are uniformly distributed on the circle. Let $x \in [0,1]$ be the point on the circle located at a distance from 0 (measured clockwise). Assume that the inverse demand function at each point x is linear and is given by $p(x) = a - bQ(x)$, where $p(x)$ is the price of the product sold at x, $Q(x)$ is the total quantity supplied at x, and $a > 0$, $b > 0$ are constants. Let $q_i(x) \in [0,\infty)$ be firm i's quantities offered at x and $Q(x) = q_1(x)$ or $q_2(x)$, because of the first-entrant-take-all rule.

Let $x_1, x_2 \in [0,1]$ be the locations of firm 1 and firm 2, respectively. Both firms have identical production and transportation technology, and they produce (and sell) a homogeneous product to the consumers. Each firm produces at a constant marginal cost, which is normalized to zero. Firm i ($i = 1,2$) bears a transport cost $t \cdot c(x, x_i)$ to ship one unit of the good from the plant (x_i) to the consumer, where $c(x, x_i)$ denotes the distance between x and x_i. Since the goods are delivered by the firms, they can discriminate across consumers. More-

bait at one endpoint. He finds that there is no subgame perfect equilibrium (SPE) with continuous location choices, while the SPE which do exist are in discrete location choices.

[5] In this paper, the location choices must be considered with the direction selections for both firms. In previous literature, a firm must consider its location and quality (in term of transport rates) based on the expected location and quality of its rival in Launhardt (1993) and Dos Santos Ferreira and Thisse (1996). In Melitz (2003), Helpman et al. (2004), firms must consider the trade-off between export and foreign direct investment to serve customers in the foreign market.

[6] If the first-entrant-take-all rule is abandoned, the current model will yield the same location equilibrium as Pal (1998) if transportation speed (the speed of new market's discovery) has nil effect on firm's profits. Therefore, this assumption is necessary in the current paper.

over, following Anderson and Neven (1991), arbitrage among consumers is assumed to be infeasible as a result of prohibitively high transport costs. In order to ensure that the whole market can always be served by both firms, it is assumed that $a > 2t$.

Each firm can only transport its product in one direction, either clockwise or counter-clockwise, until touching the other firm's territory due to the first-entrant-take-all rule. Since both firms are identical in every aspect, thus their transport speeds are identical. Let $d_1, d_2 \in \{+, -\}$ be the transport directions of firm 1 and firm 2 respectively, where "$+$" represents the clockwise direction and "$-$" is the counterclockwise one. Either firm which first arrives at point x will entirely serve the consumer and becomes a monopolist at that point. However, a special case exists when the two firms are located at the same point and deliver the goods in the same direction. It is assumed that the local markets are served by either one of the two firms in each neighboring market and thus both firms' service areas are intertwined on the circle.[7] Hence, the profits of the two firms are equal and both are half of the profits of a monopolist which serves the whole circular market.

A three stage location-direction-quantity game is constructed. The equilibrium concept adopted is a subgame perfect Nash equilibrium (SPNE), and the game will be solved by backward induction. In the first stage, the two firms simultaneously choose locations on the circle. In the second stage, after observing each other's locations, both firms simultaneously decide on the directions in which the goods are delivered, and consequently their transport distances are determined. In the third stage, both firms choose the monopoly quantities at each point that they own to maximize their profits.

By the first-entrant-take-all rule, each local market on the circle (if it could be served) will be a monopoly market and the first entrant will become a monopolist at that point. Hence, firm i's profit at each point x is

$$\pi_i(x; x_i) = \left[a - bq_i(x) - tc(x, x_i)\right] q_i(x), \quad i = 1, 2, \tag{1}$$

Solving the first order condition yields

$$q_i(x; x_i) = \frac{a - tc(x, x_i)}{2b}, \quad i = 1, 2, \tag{2}$$

and the equilibrium profit of firm i is

$$\pi_i(x; x_i) = \frac{\left[a - tc(x, x_i)\right]^2}{4b}, \quad i = 1, 2. \tag{3}$$

The equilibrium aggregate profit of firm i is the sum of the equilibrium profit on the individual market in which the firm serves. Therefore, the total profit of firm i is

$$\Pi_i(x_i) = \int_{x \in [x_i, \bar{x}_i]} \pi_i(x; x_i)\, dx, \quad i = 1, 2, \tag{4}$$

where $\bar{x}_i$ is firm i's boundary.

3. ANALYSIS

3.1 The transport direction equilibrium

Due to the symmetry of the circular market, without loss of generality, assume that $x_1 = 0$. There are four combinations of transport directions and the corresponding transport distances of the two firms:[8]

(A) $(d_1, d_2) = (+, +)$, and $c(x, x_1 = 0) = x$, $c(x, x_2) = x - x_2$,

(B) $(d_1, d_2) = (+, -)$, and $c(x, x_1 = 0) = x$, $c(x, x_2) = x_2 - x$,

(C) $(d_1, d_2) = (-, +)$, and $c(x, x_1 = 0) = 1 - x$, $c(x, x_2) = x - x_2$,

(D) $(d_1, d_2) = (-, -)$, and $c(x, x_1 = 0) = 1 - x$, $c(x, x_2) = x_2 - x$.

Two cases will be discussed separately below according to the two firms' relative locations.

Case 1: Given $x_1 = x_2 = 0$, these two firms separately choose to transport the goods either clockwise or counterclockwise. Let $\Pi_i(x_i, x_j, d_i, d_j)$ $(i \neq j)$ be firm i's total

[7] More explicitly, it can be assumed that if each local market on the circle is indexed clockwise from the origin, firm 1 may serve at the local odd markets (market 1, market 3, market 5, ⋯) and firm 2 may serve at the local even markets (market 2, market 4, market 6, ⋯).

[8] Due to the property of the circular market, it is assumed that $x_2 = 0$ when firm 2 transports the goods clockwise and $x_2 = 1$ when delivering the goods counterclockwise.

profit when firm i chooses x_i, d_i and firm j chooses x_j, d_j, respectively. If both firms choose the same location and the same direction, assume that their market are one by one and intertwined. Thus, they equally share the whole market. The total profits of firms 1 and 2 under different transport direction combinations are (When the two firms ship goods in opposite directions, each one will evenly divide the whole area that they can serve by reason of identical transportation efficiency.)

$$\Pi_1(0,0,+,+) = \Pi_2(0,0,+,+) = \frac{1}{2}\cdot\int_0^1 \frac{(a-tx)^2}{4b}dx = \frac{t^2+3a(a-t)}{12b}, \tag{5}$$

$$\Pi_1(0,0,+,-) = \Pi_2(0,0,-,+) = \int_0^{\frac{1}{2}} \frac{(a-tx)^2}{4b}dx = \frac{t^2+6a(2a-t)}{96b}, \tag{6}$$

$$\Pi_1(0,0,-,+) = \Pi_2(0,0,+,-) = \int_{\frac{1}{2}}^1 \frac{\left[a-t(1-x)\right]^2}{4b}dx = \frac{t^2+6a(2a-t)}{96b}, \tag{7}$$

and

$$\Pi_1(0,0,-,-) = \Pi_2(0,0,-,-) = \frac{1}{2}\cdot\int_0^1 \frac{\left[a-t(1-x)\right]^2}{4b}dx = \frac{t^2+3a(a-t)}{12b}. \tag{8}$$

Notice that the total profits of the two firms are the same in the $(+,+)$ and in the $(-,-)$ cases, whereas they are identical in the $(+,-)$ and in the $(-,+)$ cases. Hence, the following lemma can be verified after some simple calculations:

Lemma 1. *Given* $x_1 = x_2 = 0$, *the two firms will choose to deliver the goods in opposite directions and each firm has half of the whole market.*

Proof. Since $a > 2t$, the difference between one firm's total profit in $(+,+)$ (or $(-,-)$) and one firm's total profits in $(+,-)$ (or $(-,+)$) are as follows. Subtracting (5) (or (8)) from (6) (or (7)) yields

$$\frac{t^2+6a(2a-t)}{96b} - \frac{t^2+3a(a-t)}{12b} = \frac{t(2a-t)}{32b} > 0 \tag{9}$$

Equation (9) implies that these two firms will choose to deliver the goods in opposite directions so as to maximize their total profits.

This result indicates that if it is assumed that $x_1 = x_2 = 0$, there are two direction equilibria: $(+,-)$ and $(-,+)$. Lemma 1 is quite straightforward because each firm's average total transport distances of serving the whole circular market (in the case where $(x_1, x_2, d_1, d_2) = (0,0,+,+)$ or $(0,0,-,-)$) is longer than one firm's total transport distances of serving the half circular market (in the case where $(x_1, x_2, d_1, d_2) = (0,0,+,-)$ or (0,0,-,+)). Hence, half of the total transport costs of serving the entire circular market are larger than the total transport costs of serving half of the circular market.

Case 2: Given $x_1 = 0$, $x_2 \neq x_1$, these two firms separately choose to transport the goods either clockwise or counterclockwise. In this case, both firms are monopolists in their territories and some consumers may not be served (resulting in uncovered markets). The total profits of the two firms under various direction combinations are collected as follows.

(i) When $(+,+)$ is chosen, then

$$\Pi_1(0,x_2,+,+) = \int_0^{x_2} \frac{(a-tx)^2}{4b}dx = \frac{x_2\left[t^2x_2^2+3a(a-tx_2)\right]}{12b} \tag{10}$$

and

$$\Pi_2(0,x_2,+,+)=\int_{x_2}^{1}\frac{\left[a-t\left(x-x_2\right)\right]^2}{4b}dx$$
$$=\frac{\left(1-x_2\right)\left[t^2\left(1-x_2\right)^2+3a\left(a-t\left(1-x_2\right)\right)\right]}{12b}. \tag{11}$$

(ii) When $(+,-)$ is chosen, then

$$\Pi_1(0,x_2,+,-)=\int_{0}^{\frac{x_2}{2}}\frac{\left(a-tx\right)^2}{4b}dx=\frac{x_2\left[t^2x_2^2+6a\left(2a-tx_2\right)\right]}{96b}, \tag{12}$$

and

$$\Pi_2(0,x_2,+,-)=\int_{\frac{x_2}{2}}^{x_2}\frac{\left[a-t\left(x_2-x\right)\right]^2}{4b}dx=\frac{x_2\left[t^2x_2^2+6a\left(2a-tx_2\right)\right]}{96b}. \tag{13}$$

(iii) When $(-,+)$ is chosen, then

$$\Pi_1(0,x_2,-,+)=\int_{\frac{x_2+1}{2}}^{1}\frac{\left[a-t\left(1-x\right)\right]^2}{4b}dx$$
$$=\frac{\left(1-x_2\right)\left[t^2\left(1-x_2\right)^2+6a\left(2a-t\left(1-x_2\right)\right)\right]}{96b}. \tag{14}$$

and

$$\Pi_2(0,x_2,-,+)=\int_{x_2}^{\frac{x_2+1}{2}}\frac{\left[a-t\left(x-x_2\right)\right]^2}{4b}dx$$
$$=\frac{\left(1-x_2\right)\left[t^2\left(1-x_2\right)^2+6a\left(2a-t\left(1-x_2\right)\right)\right]}{96b}. \tag{15}$$

(iv) When $(-,-)$ is chosen, then

$$\Pi_1(0,x_2,-,-)=\int_{x_2}^{1}\frac{\left[a-t\left(1-x\right)\right]^2}{4b}dx$$
$$=\frac{\left(1-x_2\right)\left[t^2\left(1-x_2\right)^2+3a\left(a-t\left(1-x_2\right)\right)\right]}{12b}, \tag{16}$$

and

$$\Pi_2(0,x_2,-,-)=\int_{0}^{x_2}\frac{\left[a-t\left(x_2-x\right)\right]^2}{4b}dx=\frac{x_2\left[t^2x_2^2+3a\left(a-tx_2\right)\right]}{12b}. \tag{17}$$

The payoff matrix when $x_1=0$ and $x_2\neq x_1$ is presented in Table 1.

Table 1. The payoff matrix of the direction game when $x_2 \neq x_1$

Firm 1 \ Firm 2	+	−
+	$\left(\Pi_1(0,x_2,+,+),\Pi_2(0,x_2,+,+)\right)$	$\left(\Pi_1(0,x_2,+,-),\Pi_2(0,x_2,+,-)\right)$
−	$\left(\Pi_1(0,x_2,-,+),\Pi_2(0,x_2,-,+)\right)$	$\left(\Pi_1(0,x_2,-,-),\Pi_2(0,x_2,-,-)\right)$

According to four different transport direction combinations, there are four cases that will be discussed individually to obtain conditions on x_2 which will ensure the existence of an equilibrium in transport directions. Lemma 2 summarizes the results.

Lemma 2. *Given* $x_1 = 0$, $x_2 \neq x_1$,

(A) *when* $x_2 \in [1/3, 2/3]$, then $(+,+)$ *is a direction equilibrium;*

(B) *when* $x_2 \in [2/3, 1)$*, then* $(+,-)$ *is a direction equilibrium;*

(C) *when* $x_2 \in (0, 1/3]$*, then* $(-,+)$ *is a direction equilibrium*

(D) *when* $x_2 \in [1/3, 2/3]$, then $(-,-)$ *is a direction equilibrium.*

Proof. See Appendix A.

3.2 The location equilibrium

Notice that from Lemma 2 (i) and (iv), given $x_1 = 0$, $1/3 \le x_2 \le 2/3$, there are multiple transport direction combinations: $(+,+)$ and $(-,-)$ in equilibrium. The indifference location where firm 2 makes the same profits with the different direction combinations ($(+,+)$ and $(-,-)$) is explored by

$$\begin{aligned}&\Pi_2(0,x_2,+,+)-\Pi_2(0,x_2,-,-)\\&=\frac{(1-x_2)\left[t^2(1-x_2)^2+3a\left(a-t(1-x_2)\right)\right]}{12b}-\frac{x_2\left[t^2x_2^2+3a\left(a-tx_2\right)\right]}{12b}\\&=\frac{1}{12b}(1-2x_2)\left[t^2\left(x_2^2-x_2+1\right)+3a\left(a-t\right)\right].\end{aligned} \tag{18}$$

Equation (18) must be equal to zero due to an equivalent profitability under these two directional combinations. Since $0 \le x_2 \le 1$ and $a > 2t$, the contents in the bracket of equation (18) are positive. Therefore, if $x_2 = 1/2$, then $\pi_2(0,x_2,+,+) = \pi_2(0,x_2,-,-)$.

To find the best location of firm 2 under the four possible delivery equilibrium, the first order conditions (FOC, hereafter) of firm 2's profit-maximization problems with respect to x_2 should be checked in all different situations.

Case A: Given $x_1 = 0$, $1/3 \le x_2 \le 1/2$ ($1/2 \le x_2 \le 2/3$) and $(\hat{d}_1, \hat{d}_2) = (+,+)$, the FOC is

$$\frac{d\tilde{\Pi}_2(0,x_2,+,+)}{dx_2} = -\frac{\left[a-t(1-x_2)\right]^2}{4b} < 0 \tag{19}$$

Since FOC<0, it shows the location equilibrium is a corner solution. Thus, in this case, firm 2 reaches its maximal profit at $x_2 = 1/3$ ($1/2$).

Case B: Given $x_1 = 0$, $2/3 \le x_2 \le 1$, and $(\hat{d}_1, \hat{d}_2) = (+,-)$, the FOC is

$$\frac{d\tilde{\prod}_2(0,x_2,+,-)}{dx_2}=\frac{(2a-tx_2)^2}{32b}>0. \qquad (20)$$

Thus, in this case, firm 2 reaches its maximal profit at $x_2=1$.

Case C: Given $x_1=0$, $0\le x_2\le 1/3$, and $(\hat{d}_1,\hat{d}_2)=(-,+)$, the FOC is

$$\frac{d\tilde{\prod}_2(0,x_2,-,+)}{dx_2}=-\frac{\left[2a-t(1-x_2)\right]^2}{32b}<0. \qquad (21)$$

Thus, in this case, firm 2 reaches its maximal profit at $x_2=0$.

Case D: Given $x_1=0$, $1/3\le x_2\le 1/2$ ($1/2\le x_2\le 2/3$), and $(\hat{d}_1,\hat{d}_2)=(-,-)$, the FOC is

$$\frac{d\tilde{\prod}_2(0,x_2,-,-)}{dx_2}=\frac{(a-tx_2)^2}{4b}>0. \qquad (22)$$

Thus, in this case, firm 2 reaches its maximal profit at $x_2=1/2$ ($2/3$).

It can be observed from these four cases that firm 2 does not choose any intermediate location within its allowable interval. Hence, it is concluded that given $x_1=0$, firm 2 will locate at one of the four critical points: 0, $1/3$, $1/2$, $2/3$. The above results can be summarized as the following lemma.

Lemma 3. *Assume that firm 1's location is at the origin, and then firm 2 will locate at one of the critical points,* $[0,1/3,1/2,2/3]$. It is straightforward to explain this phenomenon. Given firm 1's location and transport direction equilibrium, firm 2 will choose its best location under its restricted interval to maximize its profits. Any intermediate location within the interval is not the best choice since one can always find a point which is near some endpoint to expand firm 2's market area and hence increase its total profits.

From Case 1 and Case 2, four critical points about firm 2's location are collected: $x_2=[0,1/3,1/2,2/3]$. These four critical points are the indifference points that enable the two firms to have the same profits in specific transport direction combinations or to enable firm 2 to have the same profit in either transport direction. The former situation indicates that firm 2 may choose to be set at 0 ($1/2$) so as to earn the same profit (and serve the same market area) as firm 1 if both firms choose to deliver the goods in the opposite (same) direction. While the latter means that given that firm 1 is located at the origin and ships the goods clockwise (counterclockwise), firm 2 can earn the same profit (and serve the same market area) in either transport direction when it is located at $2/3$ ($1/3$).

Combining Case 1 and Case 2, given the direction equilibrium $(\hat{d}_1,\hat{d}_2)$ under different restricted location intervals, the two firms' equilibrium profits (denoted by $\tilde{\prod}_1$ and $\tilde{\prod}_2$), given $x_1=0$, can be summarized below:

(A) If $(\hat{d}_1,\hat{d}_2)=(+,+)$, it must be that $1/3\le x_2\le 1/2$ or $1/2\le x_2\le 2/3$;[9] meanwhile, $\tilde{\prod}_1(0,x_2,+,+)$ and $\tilde{\prod}_2(0,x_2,+,+)$ are the same as (10) and (11), respectively.

(B) If $(\hat{d}_1,\hat{d}_2)=(+,-)$, it must indicate that $2/3\le x_2\le 1$; meanwhile, $\tilde{\prod}_1(0,x_2,+,-)$ and $\tilde{\prod}_2(0,x_2,+,-)$ are the same as (12) and (13), respectively.

(C) If $(\hat{d}_1,\hat{d}_2)=(-,+)$, it must be that $0\le x_2\le 1/3$; meanwhile, $\tilde{\prod}_1(0,x_2,-,+)$ and $\tilde{\prod}_2(0,x_2,-,+)$ are the same as (14) and (15), respectively.

(D) If $(\hat{d}_1,\hat{d}_2)=(-,-)$, it must imply that $1/3\le x_2\le 1/2$ or $1/2\le x_2\le 2/3$; meanwhile, $\tilde{\prod}_1(0,x_2,-,-)$ and $\tilde{\prod}_2(0,x_2,-,-)$ are the same as (16) and (17), respectively.

Observe that if the distance between these two firms is within $1/3$, then delivering the goods in opposite directions is the best choice for firms. In contrast, if these two firms are far apart (larger than $1/3$), they may choose to ship the goods in the same direction.

The next step is to explore the best location for firm 2 among these four critical points.

[9] The intervals [1/3, 2/3] should be divided into two cases since the different equilibrium transport directions can be chosen at the two sub-intervals.

Firm 2's total profits at each critical point under all possible delivery equilibrium are listed below.

(I) Given $(x_1, x_2) = (0,0)$, then $(\hat{d}_1, \hat{d}_2) = (+,-),(-,+)$, and

$$\tilde{\Pi}_2(0,0,+,-) = \tilde{\Pi}_2(0,0,-,+) = \frac{t^2 + 6a(2a-t)}{96b} \quad (23)$$

(II) Given $(x_1, x_2) = (0,1/3)$, then $(\hat{d}_1, \hat{d}_2) = (+,+),(-,+),(-,-)$, and

$$\tilde{\Pi}_2(0,\frac{1}{3},+,+) = \frac{4t^2 + 9a(3a-2t)}{162b}, \quad (24)$$

$$\tilde{\Pi}_2(0,\frac{1}{3},-,+) = \tilde{\Pi}_2(0,\frac{1}{3},-,-) = \frac{t^2 + 9a(3a-t)}{324b} \quad (25)$$

(III) Given $(x_1, x_2) = (0,1/2)$, then $(\hat{d}_1, \hat{d}_2) = (+,+),(-,-)$, and

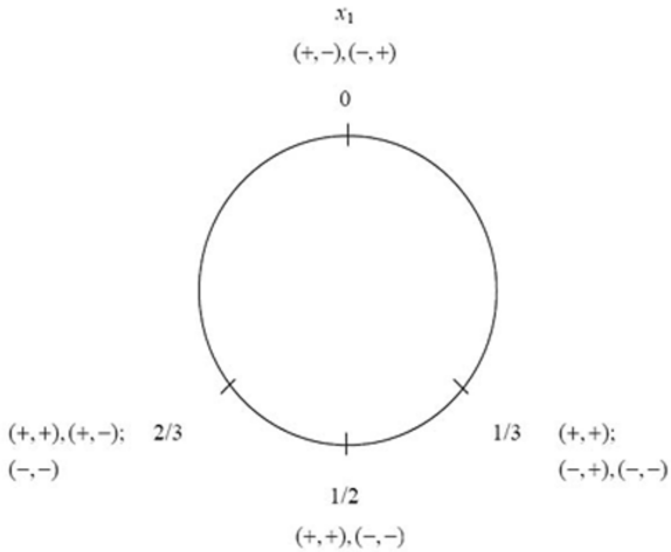
$$\tilde{\Pi}_2(0,\frac{1}{2},+,+) = \tilde{\Pi}_2(0,\frac{1}{2},-,-) = \frac{t^2 + 6a(2a-t)}{96b} \quad (26)$$

(IV) Given $(x_1, x_2) = (0,2/3)$, then $(\hat{d}_1, \hat{d}_2) = (+,+),(+,-),(-,-)$, and

$$\tilde{\Pi}_2(0,\frac{2}{3},+,+) = \tilde{\Pi}_2(0,\frac{2}{3},+,-) = \frac{t^2 + 9a(3a-t)}{324b} \quad (27)$$

$$\tilde{\Pi}_2(0,\frac{2}{3},-,-) = \frac{4t^2 + 9a(3a-2t)}{162b}. \quad (28)$$

Figure 1. The candidate locations of firm 2 and subsequent equilibrium direction combinations.



Observe that firm 2's total profits have three types ((I) (23) and (26); (II) (24) and (28); (III) (25) and (27)), and the comparison among these three types are[10]

[10] Since $a > 2t$, subtracting (23) (or (26)) from (24) (or (28)) yields

$$\tilde{\Pi}_2(0,1/3,+,+)(=\tilde{\Pi}_2(0,2/3,-,-)) - \tilde{\Pi}_2(0,0,+,-)(=\tilde{\Pi}_2(0,0,-,+)) = \tilde{\Pi}_2(0,1/2,+,+) = \tilde{\Pi}_2(0,1/2,-,-)$$

$$= \left[4t^2 + 9a(3a-2t)\right]\Big/162b - \left[t^2 + 6a(2a-t)\right]\Big/96b = \left[37t^2 + 18a(6a-7t)\right]\Big/2592b > 0,$$

and subtracting (25) (or (27)) from (23) (or (26)) yields

$$\tilde{\Pi}_2(0,0,+,-)(=\tilde{\Pi}_2(0,0,-,+) = \tilde{\Pi}_2(0,1/2,+,+) = \tilde{\Pi}_2(0,1/2,-,-)) - \tilde{\Pi}_2(0,1/3,-,+)(=\tilde{\Pi}_2(0,1/3,-,-)$$

$$= \tilde{\Pi}_2(0,2/3,+,+) = \tilde{\Pi}_2(0,2/3,+,-))$$

$$= \left[t^2 + 6a(2a-t)\right]\Big/96b - \left[t^2 + 9a(3a-t)\right]\Big/324b = \left[19t^2 + 18a(6a-5t)\right]\Big/2592b > 0.$$

$$\frac{t^2+9a(3a-t)}{324b} < \frac{t^2+6a(2a-t)}{96b} < \frac{4t^2+9a(3a-2t)}{162b} \quad (29)$$

Since $\tilde{\prod}_2(0,0,+,-)=\tilde{\prod}_2(0,0,-,+)$, the transport direction choice between $(+,-)$ and $(-,+)$ are indifferent for firm 2. Similar situations occur when

(A) given $(x_1,x_2)=(0,1/3)$, $(\hat{d}_1,\hat{d}_2)=(-,+),(-,-)$, and

(B) given $(x_1,x_2)=(0,1/2)$, $(\hat{d}_1,\hat{d}_2)=(+,+),(-,-)$, and

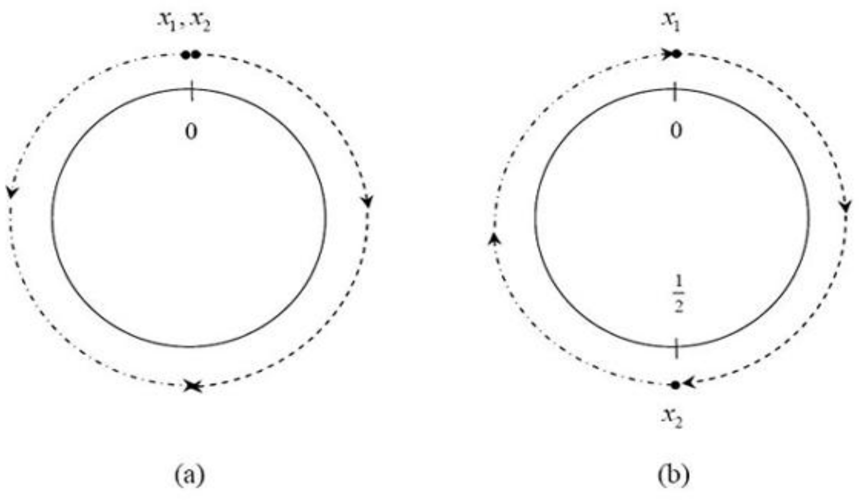
(C) given $(x_1,x_2)=(0,2/3)$, $(\hat{d}_1,\hat{d}_2)=(+,+),(+,-)$.

The discussion about firm 2's best location can be simplified and separated to four cases (see Appendix B), and Figure 1 summarizes (23) to (28) and clarifies firm 2's possible locations as well as the candidate transport direction equilibria.

After a series of comparisons (see Appendix B for details), the location-direction equilibrium can be summarized in the following proposition.

Proposition 1. *In the location-direction-quantity game, if the two firms competing in quantity with a first-entrant-take-all rule by way of delivery direction choice in a circular market, there are two equilibrium outcomes: one is that the two firms are located at the same point and deliver the goods in opposite directions* $(x_1,x_2,d_1,d_2)=(0,0,+,-)$ *or* $(0,0,-,+)$, *and the other is that the two firms are located equidistantly from each other and deliver the goods in the same direction* $(x_1,x_2,d_1,d_2)=(0,1/2,+,+)$ *or* $(0,1/2,-,-)$

Figure 2. The equilibrium outcomes of this paper: the firms' markets are non-overlapping



(a) Agglomeration result and delivery in opposite directions

(b) Dispersion result and delivery in the same direction (one of the two situations is illustrated)

The equilibrium outcomes can be presented in Figure 2. In general, there are two opposite forces in spatial competition. One is the centripetal force (or the "natural location effect") which induces a firm to find a route which is most beneficial to it without considering the influence of its rivals. There is no such strength in a circular market since no point is superior to the rest on a circle. In contrast, the midpoint is the natural strength location of a linear market. The other one is a centrifugal force (or the "strategic location effect") which is an interactive effect causing firms to be located far apart so as to avoid competition. Anderson and Neven's (1991) model can be seen as the natural location effect dominating over the strategic location effect, while in this paper it is totally reversed. When the two firms are close enough, they choose to deliver the goods along the longer arc to maximize their territories. In this situation, both firms will be located at the same point in order to maximize their profits and mitigate drastic competition. (Although $x_1 = x_2 = 0$ and $(d_1, x_2) = (+,-)$ or $(-,+)$ is an location-direction equilibrium, in some sense, it may be seen as a "maximal differentiation" instead of "minimal differentiation," because they are located back-to-back at one point and ship their goods in opposite directions, and thus the competition frontier is at the most remote location ($1/2$)).

However, if the distance between the two firms' location is remote, shipping the goods in the same direction is an obvious better choice. (An alternative view of our result is as follows. The firms may relax competition in two different ways. They can either locate far apart, so that each firm becomes a "foreign competitor" to the other firm; or they can locate at the same site, thereby becoming "local competitors." In the latter case, firms relax competition by opting to supply different markets. We thank one of the referees for this suggestion.)

Figure 3. Pal's (1998) results: the firms' markets are overlapping.

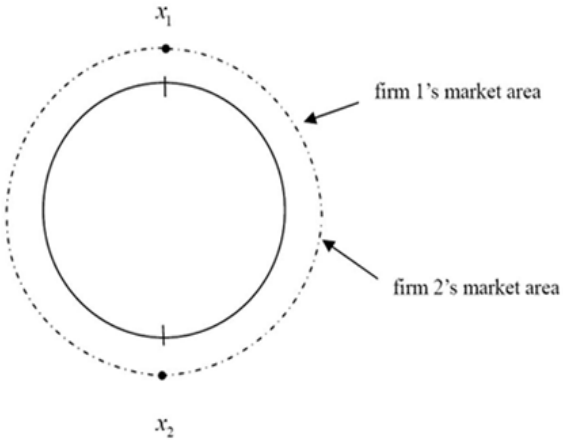


Table 2. The comparison of results between Pal (1998) and the findings from this paper

Models \ Categories	Location patterns	Directions
Pal (1998)	Maximal differentiation	None
This paper	Maximal differentiation	$(+,+)$ or $(-,-)$
	Minimal differentiation	$(+,-)$ or $(-,+)$

In summary, two new assumptions are adopted and emphasized in this paper. First, being a monopolist and having a non-overlapping service area are caused by the first-entrant-take-all rule. Secondly, unidirectional delivery results in a spatial agglomeration. Hence, Proposition 1 is very different from the results reported by Pal (1998) (see Figure 3) where firms are located equidistantly on a circle (spatial dispersion). The comparison between the results from Pal (1998) and this paper are summarized in Table 2.

4. CONCLUSIONS

Some industries, such as the oil, natural gas, cement and ready-mixed concrete industries as mentioned by Anderson and Nenen (1991) and Pal and Sarkar (2002), are often seen as natural monopolists not only by their high fixed costs (in laying out the pipelines or building production facilities), but also due to their customers' one-house-one-outlet feature. In this paper, the former characteristic is captured by a choice of delivery directions, while the latter is captured by the first-entrant-take-all rule. In contrast to the equidistant separation in Pal (1998), it is shown that spatial duopolists will be either located at one point with opposite delivery directions or be located at two endpoints of a diameter with the same delivery direction. In the real world, the Treaty of Tordesillas between Spain and Portugal perhaps can be seen as the former equilibrium, while some cross-continental airline companies cooperate in providing a global service (with each one only serving a hemisphere) may be seen as the latter equilibrium.

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APPENDIX A

Proof of Lemma 2:

Four cases which are categorized by four different transport direction combinations are discussed below to obtain the necessary conditions for x_2.

Case A.1: If $(+,+)$ is a Nash equilibrium of the direction game (please refer Table 1), then

$$\Pi_1(0,x_2,+,+) \geq \Pi_1(0,x_2,-,+)\,, \tag{A.1}$$

$$\Pi_2(0,x_2,+,+) \geq \Pi_2(0,x_2,+,-) \tag{A.2}$$

Equation (A.1) and (A.2) imply that

$$\begin{aligned}
&\Pi_1(0,x_2,+,+) - \Pi_1(0,x_2,-,+) \\
&= \frac{x_2\left[t^2x_2^2 + 3a\left(a - tx_2\right)\right]}{12b} - \frac{\left(1-x_2\right)\left[t^2\left(1-x_2\right)^2 + 6a\left(2a - t\left(1-x_2\right)\right)\right]}{96b} \\
&= \frac{1}{96b}\left(3x_2 - 1\right)\left[t^2\left(3x_2^2 + 1\right) + 6a\left(2a - t\left(1+x_2\right)\right)\right] \geq 0,
\end{aligned} \tag{A.3}$$

and

$$\begin{aligned}
&\Pi_2(0,x_2,+,+) - \Pi_2(0,x_2,+,-) \\
&= \frac{\left(1-x_2\right)\left[t^2\left(1-x_2\right)^2 + 3a\left(a - t\left(1-x_2\right)\right)\right]}{12b} - \frac{x_2\left[t^2x_2^2 + 6a\left(2a - tx_2\right)\right]}{96b} \\
&= \frac{1}{96b}\left(2 - 3x_2\right)\left[t^2\left(3x_2^2 - 6x_2 + 4\right) + 6a\left(2a - t\left(2 - x_2\right)\right)\right] \geq 0.
\end{aligned} \tag{A.4}$$

Since $0 \leq x_2 \leq 1$ and $a > 2t$, it can be checked that the contents in the brackets of the right-hand side of equation (A.3) and (A.4) are both positive. Therefore, it must be that $x_2 \geq 1/3$ in (A.3) and $x_2 \leq 2/3$ in (A.4) correspond with the requirements of (A.3) and (A.4), respectively. Therefore, $x_2 \in [1/3, 2/3]$ and given $x_1 = 0$, then $(+,+)$ is a Nash equilibrium.

Case A.2: If $(+,-)$ is a direction Nash equilibrium of this game, then

$$\Pi_1(0,x_2,+,-) \geq \Pi_1(0,x_2,-,-), \tag{A.5}$$

$$\Pi_2(0,x_2,+,-) \geq \Pi_2(0,x_2,+,+). \tag{A.6}$$

Equation (A.5) and (A.6) imply that

$$\begin{aligned}
&\Pi_1(0,x_2,+,-) - \Pi_1(0,x_2,-,-) \\
&= \Pi_2(0,x_2,+,-) - \Pi_2(0,x_2,+,+) \\
&= \frac{x_2\left[t^2x_2^2 + 6a\left(2a - tx_2\right)\right]}{96b} - \frac{\left(1-x_2\right)\left[t^2\left(1-x_2\right)^2 + 3a\left(a - t\left(1-x_2\right)\right)\right]}{12b} \\
&= \frac{1}{96b}\left(3x_2 - 2\right)\left[t^2\left(3x_2^2 - 6x_2 + 4\right) + 6a\left(2a - t\left(2 - x_2\right)\right)\right] \geq 0.
\end{aligned} \tag{A.7}$$

After a similar inference in Case A.1, it can be concluded that if $x_2 \in [2/3, 1)$ and given $x_1 = 0$, then $(+,-)$ is a direction Nash equilibrium of this game.

Case A.3: If $(-,+)$ is a Nash equilibrium of this game, then

$$\Pi_1(0,x_2,-,+) \geq \Pi_1(0,x_2,+,+), \tag{A.8}$$

$$\Pi_2(0,x_2,-,+) \geq \Pi_2(0,x_2,-,-). \tag{A.9}$$

Equation (A.8) and (A.9) imply that

$$\begin{aligned}
&\Pi_1(0,x_2,-,+)-\Pi_1(0,x_2,+,+)\\
&=\Pi_2(0,x_2,-,+)-\Pi_2(0,x_2,-,-)\\
&=\frac{(1-x_2)\left[t^2(1-x_2)^2+6a\left(2a-t(1-x_2)\right)\right]}{96b}-\frac{x_2\left[t^2x_2^2+3a\left(a-tx_2\right)\right]}{12b} \qquad \text{(A.10)}\\
&=\frac{1}{96b}(1-3x_2)\left[t^2\left(3x_2^2+1\right)+6a\left(2a-t(1+x_2)\right)\right]\geq 0.
\end{aligned}$$

Again, after a similar inference in Case A.1, it can be concluded that if $x_2\in[0,1/3]$ and given $x_1=0$, then $(-,+)$ is a (direction) Nash equilibrium.

Case A.4: If $(-,-)$ is a Nash equilibrium of this game, then

$$\Pi_1(0,x_2,-,-)\geq\Pi_1(0,x_2,+,-), \qquad \text{(A.11)}$$

$$\Pi_2(0,x_2,-,-)\geq\Pi_2(0,x_2,-,+). \qquad \text{(A.12)}$$

Equation (A.11) and (A.12) imply that

$$\begin{aligned}
&\Pi_1(0,x_2,-,-)-\Pi_1(0,x_2,+,-)\\
&=\frac{(1-x_2)\left[t^2(1-x_2)^2+3a\left(a-t(1-x_2)\right)\right]}{12b}-\frac{x_2\left[t^2x_2^2+6a\left(2a-tx_2\right)\right]}{96b} \qquad \text{(A.13)}\\
&=\frac{1}{96b}(2-3x_2)\left[t^2\left(3x_2^2-6x_2+4\right)+6a\left(2a-t(2-x_2)\right)\right]\geq 0,
\end{aligned}$$

and

$$\begin{aligned}
&\Pi_2(0,x_2,-,-)-\Pi_2(0,x_2,-,+)\\
&=\frac{x_2\left[t^2x_2^2+3a\left(a-tx_2\right)\right]}{12b}-\frac{(1-x_2)\left[t^2(1-x_2)^2+6a\left(2a-t(1-x_2)\right)\right]}{96b} \qquad \text{(A.14)}\\
&=\frac{1}{96b}(3x_2-1)\left[t^2\left(3x_2^2+1\right)+6a\left(2a-t(1+x_2)\right)\right]\geq 0,
\end{aligned}$$

which are the same as equations (A.4) and (A.3), respectively. Hence, if $x_2\in[1/3,2/3]$, and given $x_1=0$, then $(-,-)$ is a Nash equilibrium in direction.

Appendix B

Following Lemma 3, the potential location equilibrium outcome in the first stage is now reduced to four points $[0,1/3,1/2,2/3]$ (given $x_1=0$). Firm 2 can choose one of these four points as its location and he must further consider all consequent possible direction equilibria for his location choice. All the comparisons are listed in Table 1. From Figure 1, given $(d_1,d_2)=(+,-)$ and $x_1=0$, only $x_2=2/3$ can satisfy the equilibrium condition. However, from Table 1 $\pi_2(0,0,+,-)=\frac{t^2+6a(2a-t)}{96b}>\pi_2(0,\frac{2}{3},+,-)$.

Thus firm 2 will choose $x_2=0$. Furthermore, $(x_1,x_2)=(0,0)$ should satisfy the symmetry to be a location-direction equilibrium outcome. In other words, if (x_1^*,x_2^*) is a location equilibrium, then if now $x_1'=x_2^*$, it must be $x_2'=x_1^*$. In the current case, given $(d_1,d_2)=(+,-)$ and $x_2=0$, then firm 1 will also choose $x_1=0$. Therefore, $(x_1,x_2,d_1,d_2)=(0,0,+,-)$ is a location-direction equilibrium outcome.

Given $(d_1,d_2)=(-,+)$ and $x_1=0$. It is similar to prove that (x_1,x_2,d_1,d_2)

$=(0,0,-,+)$ is a location-direction equilibrium.

Given $(d_1,d_2)=(+,+)$, then $(x_1,x_2)=(0,1/3),(0,1/2),(0,2/3)$ all satisfy the equilibrium conditions. Since $\pi_2(0,\frac{1}{3},+,+)=\frac{4t^2+9a(3a-2t)}{162b}>\pi_2(0,\frac{1}{2},+,+)$ $>\pi_2(0,\frac{2}{3},+,+)$, firm 2 will choose $x_2=1/3$. However, given $x_1'=1/3$, then firm 2 will choose $x_2'=2/3$ which violates the symmetric condition. Therefore, $\pi_2(0,1/3,+,+)$ can not be a location-direction equilibrium outcome. Moreover, given $x_1^1=1/2$, then the best response for firm 2 is $x_2=0$. It also satisfies the symmetric condition and thus $(x_1,x_2,d_1,d_2)=(0,1/2,+,+)$ is a location-direction equilibrium outcome.

Similarly, given $(d_1,d_2)=(-,-)$ and $x_1=0$, then $\pi_2(0,2/3,-,-)$ $=\frac{4t^2+9a(3a-t)}{324b}$ is higher than $\pi_2(0,1/3,-,-)$ and $\pi_2(0,1/2,-,-)$. Therefore, firm 2 will choose $x_2=2/3$. However, when $x_2'=2/3$, firm 1 will choose $x_1'=1/3$ by symmetry. Therefore, $(0,2/3,-,-)$ cannot be a location-direction equilibrium outcome. However when $(d_1,d_2)=(-,-)$, if $x_1=1/2$, then $x_2=0$ by symmetry. Therefore, $(x_1,x_2,d_1,d_2)=(0,1/2,-,-,)$ is a location-direction equilibrium outcome.

Table 3. Checking the location-direction equilibria

x_1	x_2	$\hat{d}_1$	$\hat{d}_2$	Symmetry		π_2
0	0	+	−	Yes.	$(x_1'=0\Rightarrow x_2'=0=x_1)$	$\frac{t^2+6a(2a-t)}{96b}$
0	$\frac{2}{3}$	+	−	No.	$(x_1'=\frac{2}{3}\Rightarrow x_2'=\frac{1}{3}\neq 0=x_1)$	$\frac{t^2+9a(3a-t)}{324b}$
0	0	−	+	Yes.	$(x_1'=0\Rightarrow x_2'=0=x_1)$	$\frac{t^2+6a(2a-t)}{96b}$
0	$\frac{1}{3}$	−	+	No.	$(x_1'=\frac{1}{3}\Rightarrow x_2'=\frac{2}{3}\neq 0=x_1)$	$\frac{t^2+9a(3a-t)}{324b}$
0	$\frac{1}{3}$	+	+	No.	$(x_1'=\frac{1}{3}\Rightarrow x_2'=\frac{2}{3}\neq 0=x_1)$	$\frac{4t^2+9a(3a-2t)}{162b}$
0	$\frac{1}{3}$	+	+	Yes.	$(x_1'=\frac{1}{2}\Rightarrow x_2'=0=x_1)$	$\frac{t^2+6a(2a-t)}{96b}$
0	$\frac{1}{3}$	+	+	No.	$(x_1'=\frac{2}{3}\Rightarrow x_2'=\frac{1}{3}\neq 0=x_1)$	$\frac{t^2+9a(3a-t)}{324b}$
0	$\frac{1}{3}$	−	−	No.	$(x_1'=\frac{1}{3}\Rightarrow x_2'=\frac{2}{3}\neq 0=x_1)$	$\frac{t^2+9a(3a-t)}{324b}$
0	$\frac{1}{2}$	−	−	Yes.	$(x_1'=\frac{1}{2}\Rightarrow x_2'=0=x_1)$	$\frac{t^2+6a(2a-t)}{96b}$
0	$\frac{2}{3}$	−	−	No.	$(x_1'=\frac{2}{3}\Rightarrow x_2'=\frac{1}{3}\neq 0=x_1)$	$\frac{4t^2+9a(3a-2t)}{162b}$