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Alonso Meets Hansen: Rent Determinants and Threshold Effects

Alonso e Hansen: Determinantes e Limiares do Mercado de Arrendamento

João Pedro Ferreira
Joao.ferreira@ufl.edu
Food and Resource Economics Department, University of Florida, USA

Nuno Baetas da Silva
CeBER, Faculty of Economics, University of Coimbra, Portugal

Esteban Férnandez Vázquez
University of Oviedo, Spain

Abstract/ Resumo

The distance to the closest Central Business District has been, since Alonso theoretical contribution, one of the main variables used to explain housing prices. Recent studies made in the context of modern polycentric cities, with complex transit systems and heterogeneous social realities, have underline that other factors can also contribute to explain different housing prices. In this work, we evaluate if distance is still a factor that explains housing prices in the context of the Lisbon Metropolitan Area while also highlighting other explanatory factors. Also, combining Alonso's proposal with Hansen econometric techniques, we show that in the Lisbon area there are different segments and as we step away from the center, the price constantly declines but at a slowest rate. The results also highlight how physical aspects of the dwelling, location variables or socio-economic characteristics of the neighborhood matter in the definition of housing prices. Finally, this work concludes by discussing how this method can be improved and contribute for better public policy, namely in terms of zoning or property taxes differentiation.

A distância ao principal centro das cidades tem sido, desde o contributo teórico de Alonso, a característica que é recorrentemente utilizada para explicar o preço de dada habitação. Estudos mais recentes vieram demonstrar que com o surgimento de cidades multipolares, com redes de transportes complexas e uma organização territorial socialmente diferenciada, outros fatores podem assumir-se como igualmente relevantes para a determinação do preço da habitação. Este trabalho tem como objetivo avaliar-se se a distância ainda é um fator explicativo do preço das rendas na Área Metropolitana de Lisboa e quais outros fatores emergem como igualmente importantes. Adicionalmente, avalia-se a não linearidade desta relação através da aplicação da técnica econométrica proposta por Hansen. Através deste método concluímos que na área metropolitana de Lisboa existem segmentos de procura distintos e quando mais nos vamos afastando do centro mais o preço vai diminuindo de forma menos acentuada. Os resultados mostram que, tal como a distância, as características fisicas, a localização e outras características socio-económicos da freguesia interessam na definição do preço da habitação. Por fim, este trabalho discute como este método por ser melhorado e contribuir para a existência de melhores políticas públicas, em particular a aplicada à escala urbana.

Keywords: Alonso, distance, housing prices, metropolitan areas, rent, threshold y

JEL Code: R15, R31

Palavras-chave: Alonso, área metropolitana, distância, habitação, preço, renda

Código JEL: R15, R31

1. INTRODUCTION

Housing, along with its related services, represents the most important share of households' expenditures in modern societies. Housing, and all it entails, is a complex good that demands the use of advanced techniques and methods to understand its market. We can buy or rent a new or existing house, which has one or many rooms and may or may not include a garage, a fireplace, or a garden, that is located in an urban or rural area. A major problem with housing prices is that they are not constant nor easy to predict or understand (Malpezzi, 1996). The last economic crisis showed how housing prices can affect or be affected by related economic activities or unexpected events. As a result, the price of housing is still an open topic of research, particularly in the way it relates to geographical location within a territory.

Regarding the relationship between housing and the territory, Alonso's location and land use theory (1964) opened new avenues in this field and, for many years, set the groundwork for new theoretical developments. The increased availability of fine-scale data and computational capacity allowed easier application of theoretical frameworks to real world cases, with a particular focus on important metropolitan areas. Alonso's main idea departed from the concept of spatial equilibrium and the inverse relationship between housing and transportation costs in urban areas.

Through the years, real life examples about the geographical distribution of housing prices showed how reality is more complex than the simple Alonso's model and many improvements were suggested. If distance to the center usually matters, many cities have also experienced the rise of second- and third-order centers, almost as described in Christaller's (1966) center-place theory. Characteristics revealed to be important that are not homogenously distributed include amenities, zoning policies for housing, schools, and services, transportation networks, etc. In a literature review, Sirmans et al. (2005) states that Alonso's theory was tested against the reality and subjected to innumerable and distinct conclusions.

Therefore, this work, inspired by the Alonso's land-use theory, adds an innovative econometric modeling proposal and applies it in a real-life example: the Lisbon Metropolitan Area. The paper has two main goals. First, is to determine whether distance and other related variables are statistically significant when determining housing prices in the Lisbon metropolitan area, as suggested by Alonso's theoretical model. Second, is to determine whether this is a linear behavior or whether it is possible to identify any different land-use preferences according to the distance to the Central Business District. To the best of our knowledge, this paper will be the first to use a sample-split approach as suggested by Hansen (2000) for a metropolitan housing market.

This paper starts by briefly presenting Alonso's theory and some of its theoretical complements as well as other more recent works based on the relation between distance and housing prices. Next, we explain Hansen innovative approach and discuss how it can be used to improve our knowledge of housing markets. Then, two applications to the same geographical area, Lisbon Metropolitan Area, are developed. The first application benefits from the Census data (the most extensive data set at the parish level in Portugal). The second application uses more recent data but instead less variables were available to be tested. Finally, the results show that a Hansen threshold can be found in both exercises and an explanation for the coefficients and variables found statistically significant is presented. Finally, we conclude by exemplifying how this technique can be replied in order to produce relevant inputs to perform more adequate public policy and what are the improvements that can be applied in future works.

2. ALONSO'S THEORY AND ITS DEVELOPMENT

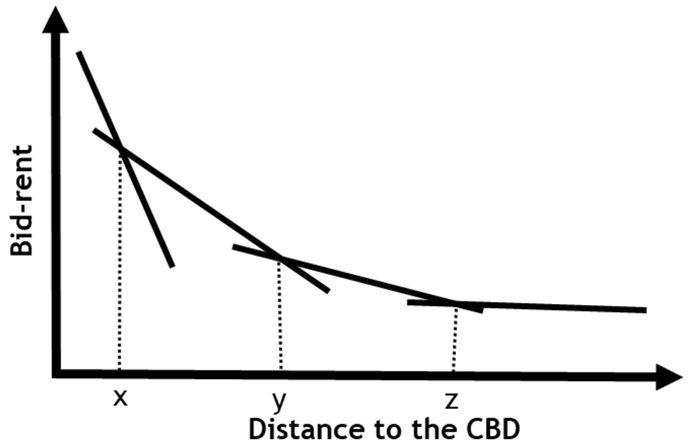
Alonso (1964) presented the first theoretical contribution to link rents and geography through distance and transportation networks. Commonly referred to as the monocentric city hypothesis, Alonso's model is based on three assumptions: 1) the city is monocentric (a single fixed job center called the central business district [CBD]), 2) there is a radial transport system between residences and workplaces, and 3) all land parcels are identical (without different proximities to public goods or influence of externalities). In a simple micro framework, a household will seek to maximize its utility, choosing a bundle of consumption and land goods, subject to a budget constraint. Assuming that the household earns a fixed income and that transportation costs increase and rents per unit of land decrease with the distance to the CBD, an equilibrium arises between the price of land (and its use) and the transportation cost. As such, housing prices are expected to be higher near the center and significantly less costly on the periphery (where the commuting costs are higher). Next, there were two landmark works that significantly improved the realism of Alonso's original model. First, Muth (1969) included the role of income in establishing urban hierarchies and argued that if a consumer with greater income has the same travel marginal cost, and has a preference for a bigger house, then he is going to prefer to live further. This could largely explain the suburbanization process of cities in the US. Additionally, Mills (1972) included a more realistic assumption on transportation costs. Nevertheless, it was open the avenue for Alonso's initial model to be improved.

The increase in data availability and the capacity to perform complex estimations has contributed to several studies on understanding the relationship established by Alonso in the different cities context. Such estimations, among other variables, examines the inclusion of distance (both measured in kilometers or in minutes) as an explanatory factor of housing rents or value. In most works, the variables influencing housing prices were assessed through the so-called econometric hedonic models. Such type of modelling assumes that products are heterogenous and can be characterized by a set of distinctive characteristics (Rosen, 1974). At the time, such technique revealed particularly interesting to the study of housing market as houses can have distinct sizes, locations, surroundings, number of rooms, building materials, number of bathrooms, having a garden or a garage, among other differentiation. So, during the 1980s, this methodology gave origin to several studies specially devoted to the analysis of housing prices (Blomquist and Worley, 1981). During the first hedonic modeling phase, the variables used were mostly associated with the physical characteristics of each dwelling.

Only a few years after, in what was a combination of Rosen hedonic models and Alonso theoretical model, distance started to be considered as a factor. As an example, while observing the reality of Australian municipalities, Kulish et al. (2012) identified a clear tendency toward higher values for land located near the CBD, as the average land value for the five suburbs near the CBD was around 16 times the average value of the suburbs located farther away from the CBD. Similarly, Ahlfeldt (2011), using OLS estimation, concludes there is a significantly negative impact of the distance to CBD proxy (2.26% per 1 km increase) in the case of the city of Berlin. Ahlfedlt (2011) points out the relevance of decentralized employment and how the inclusion of other distances can improve the estimated results. Rehak and Kacer (2019) also conclude for the existence of a negative relation between distance and housing prices in Bratislava. According to this work, the increase in distance from the city center by one percent is associated with price decrease by 0.19 percent.

By using the common Ordinary Least Squares methodology, most of these works assume a linear relation between the dependent variable (housing price) and the explanatory variables including. This was also true to distance. In other words, distance would imply a change in housing prices even if we were further or closer to city center. Alternatively, Ottensmann et al. (2008) tested a different idea and highlight the non-linear relationship between distance and housing rents using a dummy variable to control for the negative effects of the location within the Center Township. Before, Balchin et al. (1995) theoretically argued that non-linearity should complemented Alonso's more basic model if we include the assumption of distinct land use preferences. Such preferences could result from the different types of land use (urban vs. agricultural/rural) or individuals' preferences for living in a particular area (Jordaan et al., 2004). In terms of urban organization, the effect of this behavior is the increase in identifiable specific neighborhoods that concentrate people from a specific background. Figure 1 illustrates the relationship between bid-rent and distance to the CBD.

Figure 1: Bid-rent and distance to the CBD



In Figure 1, the different lines correspond to different land uses (A, B, and C). Preference for A prevails up to a distance of x kilometers from the CBD; from x to y, B is dominant; and after y kilometers, C prevails. So, different economic agents looking for distinct land uses will tend to locate nearer to or farther from the CBD according to their preferences. Ultimately, for curves A, B, and C, land use corresponds to different bid-rent gradients that should have a decreasing slope as the distance from the CBD increases because land becomes less valuable. Near the CBD, the gradient is sharp due to intense competition for very limited land and very slight in the outer zones due to the comparative abundance of land. Changes in total urban population or in its distribution among the entire region (from 0 to z) should result in changes that could affect all slopes and intersections but, then, the equilibrium should be re-established and the different preferences observed. For Balchin et al. (1995), the edge of the urban area (distance from the CBD larger than z) corresponds to the critical limit where the curve slope equals zero. In this case, the predominant land use should be agriculture activity.

The studies mentioned above support Alonso's equilibrium, nevertheless, other physical and intangible characteristics of housing or neighborhoods can be used to explain the price definition. Their parameters can be classified into three large categories: (1) physical attributes of the dwelling; (2) socioeconomic neighborhood attributes; and (3) locational attributes, including the proximity to services, amenities, transport networks, and other equipment. According to this line of research, distance to the CBD is only one of the variables that integrates the third category.

Other models conclude that some neighborhood characteristics as teacher experience (Collins and Kaplan, 2017); or violent crime rate or average education level or poverty (Delgado and Wences, 2019) also influence the housing prices. Dantan and Picard (2016) highlighted the relevance of regional distribution of household credit constraints to explain location and housing prices in Paris. Alternatively, other set of works highlighted the role of locations within the metropolitan area other than the CBD. Li et al. (2019) underlined the role of distance to bike-sharing stations and bus transit for work travel. In an innovative approach Wittowsky et al. (2020) shows that besides the physical characteristics of a place and the social context of the neighborhood, the distance to a kindergarten or school, a park, a restaurants area or the working hubs matters to determine the housing price.

Finally, Zhang et al. (2019: 73) concludes that the distance to the Wulin Square (Hanghzou economic center) matters and that housing prices are expected to decrease 0.235% if the distance from a house to the city center increases by 1%. So, in addition to the importance of the relationship between housing prices and the CBD accessibility, other dimensions also have proved to be important.

Summing up, the definition of the relationship between housing prices and distance to the CBD has three major difficulties. First, one needs to understand if the relationship between prices and distance to the CBD exists and if the metropolitan area housing market can be explained using the Alonso theoretical model. Second, one needs to understand if the relationship between housing prices and distances include different "preferences" within a specific metropolitan area. Third, one needs to control for other variables that can also influence housing prices and include them in the modelling framework.

3. METHODOLOGY: LISBON METROPOLITAN AREA AND HANSEN THRESHOLD

The Lisbon Metropolitan Area corresponds to the most dense and populated NUTS II area in Portugal. In 2011, this area was divided into 211 parishes and 18 municipalities. The parishes vary from 355 to 66,250 inhabitants (Castelo and Alqueidão-Mem Martins, respectively) and from 0.05 km^2 to 212 km^2 (Socorro and Canha, respectively). These are the smallest statistical units for which the national statistical office provides official data concerning built environment physical attributes, population statistics, and housing prices. Our proxy to the housing prices, that is available on the Portuguese Population Census, is the average rent of the most contracts celebrated in the last 5 years, by parish.[1] For this we have 211 observations, each one corresponding to a specific parish in the Lisbon Metropolitan Area[2]. Then, for the more recent year of 2018, a second model is presented with other variables including in the model. Unfortunately, due to a geographical reorganization of parishes, despite that the area covered is the same, the number of parishes in this second application is only 118, which represents a large geographical aggregation.

Nevertheless, and starting by the 2011 application and following Nelson et al. (2015), in order to solve the bias that could result from leading with different dwelling typologies and isolating the implicit price of the unit area, the variable rental housing price is then transformed in Euros per square meter. As recommended by Ahlfeldt (2011) and Ferreira et al. (2012), the distance used was the road distance between each parish[3] because Euclidian distances can bias the actual distances commuters travel. This is even more relevant if we consider that the Lisbon region has an orography characterized by important natural barriers (such as the Tagus River and the Sintra Natural Park). Downtown Lisbon was defined as being in the São Nicolau parish, in the first exercise, as this represented the most expensive square meter in Lisbon's historical downtown and presented the higher negative correlation between housing prices and distance when compared to the remain parishes of the Metropolitan Area. This area also corresponds to Lisbon most important touristic area and where the number of Airbnb's have risen in the last few years (Ferreira et al., 2020). This is, indeed, the motivation for the second application.

Considering our first and more detailed exercise, we start by testing the factors that influence housing prices. For this, we used the OLS methodology. We started by testing 35 variables and its influence in our dependent variable (rent prices by square meter). A sequential procedure was followed where we eliminated the ones without any statistical significance until only those that were significant remained. This is represented in equation (1).

[1] Portuguese Census only presents accurate statistics for rented dwellings. Because they ask for the monthly charges supported by rented or owning a house, the answers from homeowners do not directly translate the housing value and are influenced by several other factors.

[2] Albeit it does not influence the mathematical application of Hansen method, any future application of this method would benefit from disaggregated data where each observation represents a different dwelling. So, yet, Parishes are the smallest statistical geographical area in Portugal, such aggregation may hide important heterogeneity.

[3] We use the physical distances (in km) following the fastest way criteria provided by Google Maps API for each trip occurring in a working day at 8:00 a.m..

$$RENT_i = \beta_1 CBD\_DIST_i + \beta_2 ROOMS_i + \beta_3 SCHOOL_i + \beta_4 CONTRACTS_i + \beta_5 SHORE_i + \beta_6 Time\_commuting_i + v_i$$

We then estimate equation (1) assuming a possible non-linear behavior including quadratic form of distance, as shown in equation (2).

$$RENT_i = \beta_1 CBD\_DIST_i + \beta_2 CBD\_DIST_i^2 + \beta_3 ROOMS_i + \beta_4 SCHOOL_i + \beta_5 CONTRACTS_i + \beta_6 SHORE_i + \beta_7 Time\_commuting_i + v_i$$

The OLS method starts by assuming that the regressors included are exogenous and, therefore, uncorrelated with the error terms. In addition, the error terms are independent and identically distributed, with zero mean and constant variance. This means that the OLS estimator is unbiased and consistent. Nevertheless, after the model is established tests should be run to confirm that the referred assumptions are still true for the list of variables chosen[4]. The results for our variables are presented in Table 1.

Table 1: Results for simple-OLS estimation

	OLS	
	Coef. And t-statistics	**St-Error**
Constant	**2.577** (21.16)***	0.121
CBD_dist	**-0.016** (-9.253)***	0.001
CBD_dist^2	**0.001** (4.988)***	0.000
Rooms	**-0.269** (-11.431)***	0.023
School	**0.046** (9.376)***	0.005
Contracts	**0.0001** (3.213)**	0.000
Shore	**0.069** (4.194)***	0.016
Time_commuting	**-0.005** (-2.142)*	0.002
Observations	211	
R-squared	0.8897	

In this first step we validate Alonso's assumption by showing that distance is statistically significant in determining housing prices in the Lisbon Metropolitan Area even when other dimensions are included. This OLS model has a significantly high R^2 and includes four statistically significant variables. The results are presented in the first column of Table 1. The variable "Rooms", representing the average number of rooms by dwelling, presents a negative sign, meaning that as the number of rooms increase, the lower, on average, is the rental housing price. This only happens because the variable renting housing price is presented in terms of euros per square meter. So, when other variables stay stable, the average rental housing price per square meter will be lower for housing typologies with a large number of bedrooms. According to Sirmans et al. (2005), different studies found different conclusions for this variable and the economic reason is tied to the fact that in certain urban areas, a higher preference for smaller houses and a lower availability, may make them relatively expensive according to the number of rooms than when they are bigger.

[4] Our model was tested for the normality assumption using the Shapiro-Wilk method. The null hypothesis was not rejected meaning that the residuals obtained follow a normal distribution. In addition, the chi-squared distribution of the Breush-Pagan produced a statistic of 9.15. The p-value associated (0.2416) does not reject homoskedasticity.

Next, the variable "Contracts" is a proxy for the intensity of the rental housing market. The idea is that such variable shows the ratio of the number of rental housing contracts signed in the last five years when compared with the total houses in the parish. An increase in the share of new rental housing contracts celebrated in the parish also has a positive effect in the housing price. Next, other important variable that translates the average set of skills (or even, income) of the population living in the parish is the average number of school years. A one-year increase in the average number of scholar years means an increase, on average, of 4.6% by square meter in the rental housing values.

Another location dimension is also included (besides distance) that allows one to conclude that the proximity to the shore is definitely important for housing prices According to the ceteris paribus hypothesis, if all the other variables are equal, the location of a dwelling next to the shore may represent an increase of approximately 6.9% in the square meter value. Another variable that is mostly associated with the commuting time is the average commuting time. So, an additional minute in average time between housing and working leads to a decrease in housing rents in 0.5%. This conclusion, and its specifically application in the Lisbon Metropolitan Area, had already been advanced in Ferreira et al. (2017) and Ferreira et al. (2018). Using Alonso's seminal contribution, it is important to highlight that despite the inclusion of all five variables, the average distance to the CBD continues to be a statistically significant variable that contributes to explain the rental housing values. Indeed, for each kilometer that a dwelling is farther from the Lisbon CBD, there is a decrease of 1.6% per square meter in the average rent paid to the landlord. Also, the inclusion of this variable in its quadratic form also raises the possibility of an existing non-linear relationship between prices and rents meaning that further developments should be addressed in order to appropriately conclude the nature of such relation.

3.1 Hansen Sample Splitting

After testing the relationship between rent and distance to the CBD using an OLS model, our objective consisted of testing whether there is any relevant threshold that reflects the existence of distinct land-use consumption preferences. So, the next step in the estimation procedure was to apply the Hansen (2000) econometric technique, developed with the goal to address the question raised by Balchin et al. (1995) and the possibility of existing different land-use consumption preferences that influence the housing price (or rent) distribution. This method is based on a sample-split or threshold regression model of the form

$$y_i = \theta_1' x_i + e_i, \;\; q_i \leq \gamma$$
$$y_i = \theta_2' x_i + e_i, \;\; q_i \leq \gamma$$

where y_i is the dependent variable, the q_i threshold variable, both real-valued, and x_i is an m-vector of the dependent variables, for $i = 1, \ldots, n$. In this model, observations may fall into two classes or regimes that depend on an unknown value of the observed variable q_i, which may be an element of x_i, that is assumed to have a continuous distribution. The seminal contribution of Hansen (2000) allows one to estimate and make valid statistical inferences on the threshold parameter γ. Denoting $\theta_2 - \theta_1$ as the threshold effect, the proposition is that as n becomes larger, the threshold effect tends to be zero, leaving the change-point estimate free of nuisance parameters.

Using the threshold parameter γ as an indicator function, it is possible to write the model $(1) - (2)$ in a single equation to obtain the regression parameters $(\theta_2, \theta_2 - \theta_1, \gamma)$. As proposed by Chan (1993) and Hansen (2000), we can obtain the least squares (LS) estimate of $\hat{\gamma}$ as the value that minimizes the concentrated sum of squared errors, $S_n(\gamma)$. Since, conditional on γ, the model is linear in θ_2 and $\theta_2 - \theta_1$, the concentrated sum of the squared errors is only a function of γ, and OLS becomes appropriate. Therefore, $\hat{\gamma}$ is the value that minimizes $S_n(\gamma)$ and is obtained by a step procedure by successively fixing the threshold variable q_i. Once $\hat{\gamma}$ is found, the slope estimates are given by $\hat{\theta}_2 = \hat{\theta}_2\,(\hat{\gamma})$ and $(\widehat{\theta_2 - \theta_1}) = (\widehat{\theta_2 - \theta_1})\,(\hat{\gamma})$.

To investigate whether the threshold is significant, we test the null hypothesis $H_0 = \theta_2 - \theta_1 = 0$ against $H_1 = \theta_2 - \theta_1 \neq 0$. That is, we test the null hypothesis of no threshold effect under which the parameter γ is not identified, meaning that we cannot read the critical values from standard distribution tables. One way to overcome this complication is by simulation, approximating the asymptotic null distribution. This procedure was proposed by Hansen (1996), where the p-values are obtained by a heteroskedastic-consistent Lagrange multiplier (LM) statistic and a bootstrap analog is used to compute the p-values.

Provided there is evidence in support of a threshold effect to obtain confidence intervals for the parameters, a common method is to invert the Wald or t-statistics. However, Dufour (1997) argues that the t or Wald statistic behaves poorly when the parameter space contains a region where identification fails, which is the case of γ when $\theta_2 - \theta_1 = 0$. Hansen (2000) suggests the confidence interval for the threshold parameter inverted from the likelihood ratio statistic. Under the assumption that the error term e_i is iid $N(0, \sigma^2)$, we may define

$$LR_n(\gamma) = n\frac{S_n(\gamma) - S_n(\hat{\gamma})}{S_n(\hat{\gamma})}$$

and the resulting confidence region is the set,

$$\hat{\Gamma} = \{\gamma : LR_n(\gamma) \leq c(\alpha)\}$$

where $c(\alpha)$ is the upper α-critical value of the non-standard asymptotic distribution of (3). If the homoscedastic assumption does not hold, Hansen (2000) defines a scaled likelihood ratio statistic given by,

$$LR_n^*(\gamma) = \frac{LR_n(\gamma)}{\hat{\eta}^2} = \frac{S_n(\gamma) - S_n(\hat{\gamma})}{\hat{\sigma}^2\hat{\eta}^2}$$

where η^2 is a nuisance parameter to be estimated. Let $\widehat{\tau_{1\iota}} = ([\widehat{\theta_2 - \theta}_1]'x_i)^2(\frac{\hat{\eta}^2}{\hat{\sigma}^2})$ and $\widehat{\tau_{2\iota}} = ([\widehat{\theta_2 - \theta}_1]'x_i)^2$. Hansen (2000) suggests the local-constant estimator,

$$\hat{\eta}^2 = \frac{\sum_{j=1}^{n} h^{-1}K_h(\frac{\hat{\gamma} - q_i}{h})\widehat{\tau_{1\iota}}}{\sum_{j=1}^{n} h^{-1}K_h(\frac{\hat{\gamma} - q_i}{h})\widehat{\tau_{2\iota}}}$$

for some bandwidth h and kernel K(.). Racine (2008) presents a discussion on bandwidth and kernel selection and performance. We may now define the amended confidence region,

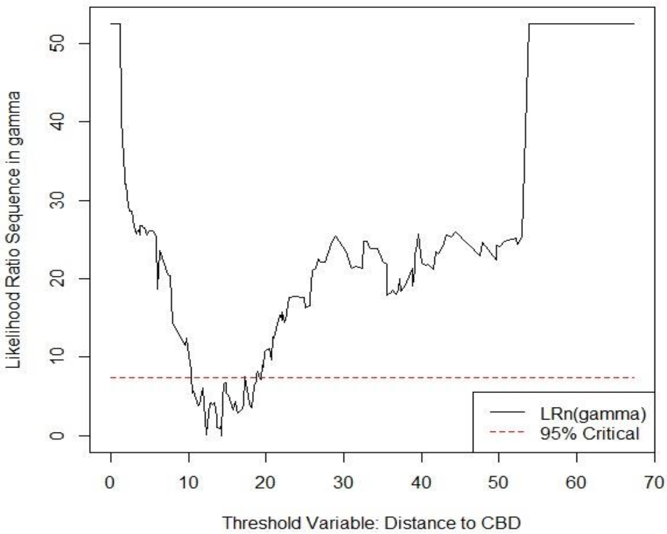
$$\hat{\Gamma} = \{\gamma : LR_n^*(\gamma) \leq c(\alpha)\}.$$

We use heteroskedastic-consistent procedures because there is evidence that the error term is heteroskedastic. The nuisance parameter η^2 is estimated using an Epanechnikov kernel with a plug-in bandwidth to avoid the inclusion of spurious noise in the density estimate. Since the threshold parameter γ is not identified under the null of no threshold effect, a bootstrap analog is used to compute the p-values. We do so by calculating the variance-covariance matrix in each replication, which results in a better finite sample approximation (Hansen, 2000). Using 1000 bootstrap replications, the p-value for the heteroskedastic-consistent Lagrange multiplier (LM) test of Hansen (1996) was significant at 0.001. There is evidence for a sample split based on CBD. These results are presented in Table 2 and Figure 1.

Table 2: Hansen model applied to the Lisbon Metropolitan Area housing market

	Threshold Model	
	Regime 1	**Regime 2**
	CBD <= 14.3	**CBD > 14.3**
Constant	**2.624**	**2.553**
	(15.5)***	(15.3)***
CBD_dist	**-0.015**	**-0.005**
	(-5.85)***	(-5.58)***
Rooms	**-0.164**	**-0.312**
	(-5.16)***	(-10.1)***
School	**0.0125**	**0.0504**
	(1.58)	(7.35)***
Contracts	**0.0001**	**0.001**
	(2.62)**	(2.67)***
Shore	**0.0329**	**0.084**
	(1.38)	(3.55)***
Time_commuting	**-0.0094**	**-0.0112**
	(-2.41)**	(-2.3)***
Observations	75	136
R-squared	0.756	0.763
Joint R-squared	0.8997	

Figure 2: Sample split: confidence interval construction for threshold



Overall, this new model presents a higher R^2 than the previous OLS model. But, in each segment, the R^2 presents a lower level, yet is significant. Figure 1 plots the normalized likelihood sequence (5) as a function of the threshold in the district distance to the CBD. The red dashed line corresponds to the 95% critical value of 7.35, computed by Hansen (2000). The LS estimate of the threshold parameter occurs at $\hat{\gamma}$=14.3, which is the value that minimizes $LR_n^*(\gamma)$. The asymptotic 95% confidence set is [10.5, 19.4]. As with the LM test, these results suggest that there might be a two-regime specification on the relationship between housing rents and the district distance to the CBD. Depending on whether the threshold variable is smaller or greater than 14.3 kilometers, each region falls into a regime. In Table 2, the estimated OLS shows the threshold regression and the difference between the coefficients. Regime 1 includes all the districts within the circle defined by the CBD with a 14.3 kilometers radius, which we label as the inner ring. Every other parish falls into Regime 2, the outside ring. Distance to CBD has a statistically significant negative sign on rents as theoretically expected. Also, the decelerating speed of rental housing prices is less sharp within Regime 2 and, indeed, the variable distance is not statistically significant in this case. To better understand the results obtained, in Regime 1, the mean rent price on average drops 1.5% when moving 1 kilometer farther away from the CDB, while in Regime 2 the mean rent price only declines 0.5% when moving 1 kilometer away from the CBD. "Rooms" variable has a negative sign in both regimes. This means that when a house has three rooms instead of two, and nothing else changes, its price per square meter, on average, is lower. As expected, this effect is stronger in Regime 2, meaning that the price of each additional room in the inner circle is higher while bigger houses outside the center can have a relatively lower price. School years variable is only significant in the Regime 2, meaning that in the periphery, housing renting price is expected to rise as the qualifications of people living in the parish also increase. Being next to the shore is also an explanatory factor of housing renting prices when houses are away from the CBD (Regime 2). The coefficient for "contracts" means that the number of new contracts celebrated in the last 5 years in the area increase, then higher will be the average rent there. This is true in both regimes. Finally, an increasing in commuting time, considering that the distance captures the effect of congestion and basically says that rents are lower in places where, for the same distance, people tend to take more time to commute.

3.2 An updated version with guesthouses impact

Next, and to double-test the flexibility and robustness of Hansen method, we decided to perform a new application. So, using the same geographical area (Lisbon Metropolitan Area), we included more recent numbers for rent market and incorporated the variables available plus the impacts of guesthouse boom. This phenomenon is detailed in Ferreira et al. (2020). For this, we use the same method but had to deal with a small number of observations (parishes) included in the analysis. As mentioned before, after some legislative transformation, the number of parishes decreased in 2013 for 118, instead of the 221 used in the previous exercise. Also, the number of variables available now (Appendix 2) are far less than the ones available for 2011 at the time of Portuguese Census (Appendix 1). Nevertheless, our goal was to replicate Hansen's method and try to conclude for a threshold that could identify distinct relations between price and distance. The results are presented in Table 3.

Table 3: Hansen model for 2018

	Threshold Model	
	Regime 1	**Regime 2**
	CBD <= 13.3	**CBD > 13.3**
Constant	**2.3485**	**1.888**
	(21.5)***	(30.48)***
CBD_dist	**-0.0091**	**-0.013**
	(-0.746)	(-9.77)***
Contracts/Dwellings	**-0.6821**	**8.570**
	(-0.5)	(3.56)***
Shore	**-0.4005**	**0.161**
	(-1.72)*	(2.28)**
Guesthouses/Dwellings	**0.5411**	**2.741**
	(1.949)*	(2.02)**
Observations	29	136
R-squared	0.424	0.763
Joint R-squared	0.832	

Using 1000 bootstrap replications, the p-value for the heteroskedastic-consistent Lagrange multiplier (LM) test of Hansen (1996) was significant at 0.01. There is evidence for a sample split based on CBD. The LS estimate of the threshold parameter occurs at $\hat{\gamma}$=13.3, which is the value that minimizes $LR_n^*(\gamma)$. The asymptotic 95% confidence set is [13.2, 14.6]. Accordingly, in the first regime (less than 13.3 kilometers from the CBD) are included 29 observations (parishes). In this case, the distance, despite the negative coefficient, is not statistically significant. In fact, only houses that are located near the shore or the presence of a relative important concentration of guesthouses are significant to explain the house renting values. Also, 90% of the observations are within Lisbon municipality. Alternatively, in the second regime, for distances further than 13.3, all the variables are statistically significant. As we may see, house renting values decrease with distance. Again, and even though the distance is not statistically significant in Regime 1, when the distance increases, in Regime 2, the mean rent price declines 1.3% for each kilometer we move further of the CBD. Additionally, an increase with a higher dynamic in renting market (variable contracts) will result in higher prices. Next, the houses located next to the shore are also expected to have a higher value, for a distance higher than 13.3 kms. Finally, and really relevant in terms of policy-making process, guesthouse concentration tends to positively influence the price of renting, namely within Lisbon municipality (Regime 1). The coefficient for the guesthouses is positive and statistically significant in both regimes. One may attribute this effect due to the shrinkage of available houses for renting or to the alternative usage of houses that is common in those specific parishes or to what is commonly mentioned as speculation (Fernandes et al., 2019; Petruzzi et al., 2020). As an example, in Regime 1, the evidence suggests that if the ratio of guesthouses per total dwelling increases by 1% then, rents per square meter will increase 1.72%[5]. Nevertheless, for the purpose of this paper, besides the analysis of each variable and its coefficient, the most important finding is that a new threshold is found. The downside is that the difference between R-Square in both regimes is now significant, which can be related with the decrease in the disaggregation level of the data or basically, the change in circumstances. Nevertheless, Hansen method is proved to be useful again.

4. CONCLUSION

This article has two main goals. The first goal is to determine whether the Alonso theory still applies in the case of the Lisbon Metropolitan Area even when other attributes are considered. The second goal is to test whether the relationship between distance and housing price includes different revealed preferences within the metropolitan region. Both goals were accomplished and after finding a negative relationship between distance to the Lisbon CBD and housing prices, the application of the Hansen methodology helped to uncover distinctive regimes in both applications. Other locational and socioeconomic variables were proved to be relevant. Also, the recent guesthouse boom is definitely related with the increase housing prices within the Lisbon Metropolitan Area, particularly those inside Lisbon municipality. Nevertheless, the applications for 2011 data and for 2018 data highlight that the quality of the models is subject to three other factors: (1) the disaggregation level of data; (2) the temporal analysis; and, (3) the independent variables that we can include in the model. Accordingly, as any other econometric modelling, an effort should be made to have a larger number of observations and time periods than can help to increase the robustness of the exercise.

The application of the Hansen methodology seems to have a whole new potential that can be further explored in housing studies. As theoretically expected, a first regime, with a higher negative slope, shows a higher sensitivity to price when houses are located near the CBD. In the second regime, the slope continues to be negative but the slope decreases, meaning that people would prefer extensive use of the land until the point where the distance to the CBD does not matter anymore and agriculture prevails. Additionally, coefficients from other variables, which were relevant in the case of the simple-OLS model, also are revealed to be statistically significant but with different coefficients. This fact can also open other avenues for further studies in the future. This model reveals an important capacity to explain price within a metropolitan area with a joint R^2 of 0.91 in the first case and a joint R^2 of 0.87 in the second and a different set of variables being proved to have statistically significance.

[5] The reason for this specific increase, is that e^(0.55)/100=1.72%.

Finally, other relevant aspect of this work consists in, by identifying the areas with distinct land uses, they also allow to understand where economic agents are willing to pay more or less for square meter. Though this, it contributes to design better public policies and instruments, namely those related with zoning policy or property taxes. In summary, distances matter and distances to the historical center have proved to be, in the case of the Lisbon Metropolitan Area, one of the most important variables. The neighboring characteristics also matter, as the average qualifications of those living in the parish is also positively related with the rental housing prices inside the geographical regions. This is also the case of the rental housing market dynamic and the number of rooms. This analysis is useful to understand how housing prices are differently affected by geography and distances, and how new infrastructures (like roads or bus lanes) or significant changes in urban forms (like the rise of second-order centers) that contribute to concentrated or sprawling urban areas can really shape the reality of housing prices.

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APPENDIX 1

Model 1: data for year 2011 and 211 parishes in lisbon metropolitan area

		Designation	Average	Median	St. Error	Min	Max
1	Average Rent*	Rent_m2	422.1	408.1	92.2	254.0	683.7
2	Rental contracts celebrated*		570.2	438.0	495.8	10.0	2693.0
3	Distance to the CBD (kms)	CBD_dist	22.8	19.4	16.0	0.0	67.4
4	Distance to the CBD (minutes)		29.6	30.0	12.1	0.0	56.0
5	Building Age		45.0	39.7	18.4	18.5	107.1
6	Number of rooms	Rooms	4.6	4.6	0.4	3.3	5.7
7	% of people living in overload houses		12.7	11.9	4.2	4.4	26.6
8	Household/km2		2900.9	1639.0	3556.1	5.7	21943.3
9	Population/km2		4769.8	2987.0	5239.9	8.0	29495.4
10	% of inhabitants who out-commute		35.8	41.0	16.4	5.4	66.7
11	Time spent by commuters	Time_commuting	25.6	25.5	3.4	17.5	35.2
12	Area		96.6	96.0	17.1	41.8	137.3
13	Unemployment rate		0.1	0.1	0.0	0.1	0.3
14	% of in-migrants in the last 5 years		12.2	11.6	4.0	4.9	29.0
15	% of population with higher education		20.5	17.6	11.9	4.0	56.5
16	Number of jobs		3795.1	2190.0	5269.9	22.0	46579.0
17	Jobs/km2		2264.2	475.2	4651.0	1.5	27457.1
18	Shore (dummy)	Shore	0.2	0.0	0.4	0.0	1.0
19	% houses without shower		1.4%	1.0%	1.2	0.0%	6.0%
20	% houses with central heating systems		7.3%	6.0%	5.1	0.0%	25.0%
21	% houses with AC		11.7%	11.0%	6.9	1.0%	43.0%
22	% houses with any heating systems		15.9%	15.0%	6.0	5.0%	43.0%
23	% shacks		0.1%	0.0%	0.6	0.0%	8.0%
24	% of houses in sharing schemes		2.9%	2.0%	2.9	0.0%	17.0%
25	NEW Rented houses/km2*	Contracts	310.9	124.0	522.3	0.3	4320.0
26	% rented houses		30.6%	26.1%	17.0	7.9%	83.6%
27	% rented houses without social housing		28.2%	23.9%	16.4	6.5%	80.1%
28	% population that left school early		2.0	1.8	1.1	0.5	8.7
29	% empty houses		6.0%	5.0%	3.5	1.0%	27.0%
30	% of houses available for sale		2.8%	2.0%	1.7	0.0%	11.0%
31	% of houses available for renting		2.8%	2.0%	2.9	0.0%	22.0%
32	% houses for demolition		0.3%	0.0%	0.7	0.0%	5.0%
33	Average years spent in school	School	10.4	10.3	1.9	6.5	15.5
34	% of social valued workers		27.3	23.8	13.3	7.6	62.8
35	South (Tagus left margin)		0.3	0.0	0.4	0.0	1.0
36	Rent/m2*		4.5	4.3	1.2	2.5	8.5

*specifically related with the contracts celebrated in the 5 years previous the 2011 Portuguese Census.

APPENDIX 2

Model 1: data for year 2018 and 118 parishes in lisbon metropolitan area

	Designation	**Average**	**Median**	**St. Error**	**Min**	**Max**
Rent/m2	Rent_m2	6.52	5.60	2.44	3.7	13.1
Distance to the CBD (kms)	CBD_dist	25.1	22.0	15.6	0	67.4
Distance to the CBD (minutes)		31.7	31.8	11.0	0	56
Guesthouses		229.4	20.0	639.1	0	4772
Contracts*		224.9	198.5	171.8	15	832
Shore	Shore	0.16	0	0.37	0	1
Dwellings		12635	10799	8475	1045	38842
Guesthouses/Dwellings	Guesthouses/Dwellings	2.03%	1.97%	5.8	0	43.7%
Contracts/Dwellings*	Contracts/Dwellings	1.78%	1.72%	0.9	0.01%	7.65%

* number of contracts celebrated in the last 5 years

APPENDIX 3

Correlation matrix of the variables tested and applied in Model 1 – the numbers on the first row and column identify the variables described in Appendix 1.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	1.0	0.3	-0.6	-0.5	0.3	0.3	-0.3	0.4	0.4	-0.3	-0.4	0.2	-0.4	0.3	0.9	0.5	0.6	0.1	-0.4	0.4	0.3	-0.2	-0.2	0.4	0.3	0.3	0.3	-0.2	0.2	0.0	0.2	-0.1	0.9	0.9	-0.4	0.7
2	0.3	1.0	-0.2	-0.2	-0.1	-0.2	0.2	0.1	0.2	0.1	0.2	-0.2	0.2	0.0	0.1	0.4	-0.1	0.2	-0.4	0.0	0.0	0.0	-0.1	0.0	0.1	0.0	0.0	-0.1	-0.2	0.0	-0.1	-0.3	0.2	0.1	-0.1	0.3
3	-0.6	-0.2	1.0	0.9	-0.5	0.3	-0.2	-0.7	-0.6	0.3	0.1	0.5	-0.1	-0.2	-0.5	-0.3	-0.5	0.1	0.2	0.1	0.1	-0.3	0.0	-0.4	-0.6	-0.7	-0.6	0.0	-0.2	0.2	-0.4	0.1	-0.5	-0.5	0.5	-0.8
4	-0.5	-0.2	0.9	1.0	-0.6	0.3	-0.3	-0.7	-0.7	0.4	0.1	0.5	-0.1	-0.2	-0.4	-0.2	-0.6	0.1	0.1	0.2	0.1	-0.4	0.0	-0.5	-0.6	-0.7	-0.7	-0.1	-0.3	0.2	-0.5	0.0	-0.4	-0.4	0.4	-0.8
5	0.3	-0.1	-0.5	-0.6	1.0	-0.2	0.1	0.7	0.5	-0.7	-0.4	-0.5	0.0	0.1	0.3	0.0	0.5	0.1	0.4	-0.2	-0.3	0.4	0.0	0.5	0.7	0.8	0.8	0.1	0.5	-0.2	0.6	0.3	0.2	0.4	-0.2	0.7
6	0.3	-0.2	0.3	0.3	-0.2	1.0	-0.8	-0.4	-0.4	0.0	-0.3	0.9	-0.5	0.1	0.4	0.2	0.2	0.0	-0.2	0.5	0.4	-0.6	-0.1	0.0	-0.4	-0.4	-0.4	-0.2	0.0	0.2	-0.1	0.1	0.4	0.4	0.1	-0.4
7	-0.3	0.2	-0.2	-0.3	0.1	-0.8	1.0	0.3	0.3	0.0	0.3	-0.7	0.6	-0.1	-0.5	-0.2	-0.1	0.0	0.2	-0.5	-0.6	0.6	0.1	0.1	0.3	0.4	0.3	0.3	0.0	-0.3	0.1	0.0	-0.5	-0.5	-0.2	0.3
8	0.4	0.1	-0.7	-0.7	0.7	-0.4	0.3	1.0	1.0	-0.4	-0.2	-0.5	0.1	0.2	0.3	0.1	0.5	-0.1	0.0	-0.3	-0.2	0.4	-0.1	0.5	1.0	0.7	0.7	0.1	0.3	-0.3	0.5	0.0	0.3	0.4	-0.3	0.8
9	0.4	0.2	-0.6	-0.7	0.5	-0.4	0.3	1.0	1.0	-0.3	-0.1	-0.5	0.2	0.1	0.3	0.1	0.4	-0.1	-0.1	-0.2	-0.2	0.3	-0.1	0.4	0.9	0.6	0.6	0.0	0.2	-0.3	0.4	-0.1	0.3	0.3	-0.3	0.8
10	-0.3	0.1	0.3	0.4	-0.7	0.0	0.0	-0.4	-0.3	1.0	0.6	0.2	0.1	0.2	-0.3	-0.2	-0.5	-0.2	-0.3	0.2	0.2	-0.3	0.0	-0.4	-0.4	-0.7	-0.6	-0.1	-0.4	0.2	-0.5	-0.2	-0.2	-0.4	0.2	-0.5
11	-0.4	0.2	0.1	0.1	-0.4	-0.3	0.3	-0.2	-0.1	0.6	1.0	-0.1	0.4	0.0	-0.4	-0.3	-0.4	0.0	-0.2	-0.2	0.0	0.1	0.1	-0.2	-0.2	-0.3	-0.3	0.0	-0.2	0.1	-0.3	-0.2	-0.3	-0.5	0.3	-0.2
12	0.2	-0.2	0.5	0.5	-0.5	0.9	-0.7	-0.5	-0.5	0.2	-0.1	1.0	-0.6	0.1	0.3	0.1	-0.1	0.0	-0.2	0.6	0.5	-0.7	-0.1	-0.2	-0.6	-0.7	-0.6	-0.2	-0.1	0.3	-0.3	0.0	0.3	0.3	0.1	-0.6
13	-0.4	0.2	-0.1	-0.1	0.0	-0.5	0.6	0.1	0.2	0.1	0.4	-0.6	1.0	-0.3	-0.5	-0.2	-0.2	0.0	0.1	-0.6	-0.3	0.5	0.1	0.0	0.1	0.3	0.2	0.3	0.0	-0.2	0.1	-0.1	-0.5	-0.5	0.3	0.0
14	0.3	0.0	-0.2	-0.2	0.1	0.1	-0.1	0.2	0.1	0.2	0.0	0.1	-0.3	1.0	0.3	0.0	0.3	0.0	0.0	0.4	0.2	0.0	-0.1	0.4	0.2	0.1	0.2	0.1	0.4	0.4	0.1	0.1	0.3	0.2	-0.1	0.2
15	0.9	0.1	-0.5	-0.4	0.3	0.4	-0.5	0.3	0.3	-0.3	-0.4	0.3	-0.5	0.3	1.0	0.5	0.6	0.1	-0.3	0.5	0.4	-0.3	-0.2	0.2	0.2	0.2	0.2	-0.3	0.2	0.1	0.2	-0.2	1.0	1.0	-0.3	0.5
16	0.5	0.4	-0.3	-0.2	0.0	0.2	-0.2	0.1	0.1	-0.2	-0.3	0.1	-0.2	0.0	0.5	1.0	0.4	0.1	-0.3	0.3	0.2	-0.2	-0.1	0.1	0.0	0.0	0.0	-0.2	-0.1	0.0	-0.1	-0.2	0.5	0.4	-0.2	0.2
17	0.6	-0.1	-0.5	-0.6	0.5	0.2	-0.1	0.5	0.4	-0.5	-0.4	-0.1	-0.2	0.3	0.6	0.4	1.0	-0.1	0.0	0.1	0.1	0.1	0.0	0.6	0.4	0.5	0.5	0.0	0.4	-0.1	0.4	0.2	0.5	0.6	-0.3	0.5
18	0.1	0.2	0.1	0.1	0.1	0.0	0.0	-0.1	-0.1	-0.2	0.0	0.0	0.0	0.0	0.1	0.1	-0.1	1.0	0.0	0.1	-0.2	0.1	0.1	0.0	-0.1	0.1	0.1	-0.1	0.0	0.0	0.0	-0.1	0.1	0.1	0.0	0.1
19	-0.4	-0.4	0.2	0.1	0.4	-0.2	0.2	0.0	-0.1	-0.3	-0.2	-0.2	0.1	0.0	-0.3	-0.3	0.0	0.0	1.0	-0.3	-0.3	0.3	0.2	0.1	0.2	0.3	0.3	0.3	0.3	0.0	0.3	0.4	-0.5	-0.2	0.1	-0.1
20	0.4	0.0	0.1	0.2	-0.2	0.5	-0.5	-0.3	-0.2	0.2	-0.2	0.6	-0.6	0.4	0.5	0.3	0.1	0.1	-0.3	1.0	0.4	-0.5	-0.1	-0.2	-0.3	-0.3	-0.3	-0.3	0.1	0.4	-0.2	-0.1	0.5	0.5	-0.2	-0.1
21	0.3	0.0	0.1	0.1	-0.3	0.4	-0.6	-0.2	-0.2	0.2	0.0	0.5	-0.3	0.2	0.4	0.2	0.1	-0.2	-0.3	0.4	1.0	-0.6	-0.1	-0.1	-0.3	-0.4	-0.3	-0.1	-0.1	0.4	-0.3	-0.1	0.4	0.4	0.5	-0.2
22	-0.2	0.0	-0.3	-0.4	0.4	-0.6	0.6	0.4	0.3	-0.3	0.1	-0.7	0.5	0.0	-0.3	-0.2	0.1	0.1	0.3	-0.5	-0.6	1.0	0.1	0.3	0.4	0.6	0.6	0.3	0.3	-0.2	0.4	0.1	-0.4	-0.3	-0.2	0.4
23	-0.2	-0.1	0.0	0.0	0.0	-0.1	0.1	-0.1	-0.1	0.0	0.1	-0.1	0.1	-0.1	-0.2	-0.1	0.0	0.1	0.2	-0.1	-0.1	0.1	1.0	0.0	-0.1	0.0	0.0	0.1	0.0	-0.1	0.0	0.0	-0.2	-0.1	0.1	-0.1
24	0.4	0.0	-0.4	-0.5	0.5	0.0	0.1	0.5	0.4	-0.4	-0.2	-0.2	0.0	0.4	0.2	0.1	0.6	0.0	0.1	-0.2	-0.1	0.3	0.0	1.0	0.4	0.5	0.5	0.2	0.4	-0.1	0.4	0.3	0.2	0.2	-0.1	0.5
25	0.3	0.1	-0.6	-0.6	0.7	-0.4	0.3	1.0	0.9	-0.4	-0.2	-0.6	0.1	0.2	0.2	0.0	0.4	-0.1	0.2	-0.3	-0.3	0.4	-0.1	0.4	1.0	0.7	0.7	0.1	0.3	-0.3	0.5	0.1	0.1	0.3	-0.3	0.7
26	0.3	0.0	-0.7	-0.7	0.8	-0.4	0.4	0.7	0.6	-0.7	-0.3	-0.7	0.3	0.1	0.2	0.0	0.5	0.1	0.3	-0.3	-0.4	0.6	0.0	0.5	0.7	1.0	1.0	0.1	0.5	-0.2	0.7	0.2	0.1	0.3	-0.3	0.8
27	0.3	0.0	-0.6	-0.7	0.8	-0.4	0.3	0.7	0.6	-0.6	-0.3	-0.6	0.2	0.2	0.2	0.0	0.5	0.1	0.3	-0.3	-0.3	0.6	0.0	0.5	0.7	1.0	1.0	0.1	0.6	-0.2	0.7	0.2	0.1	0.3	-0.3	0.8
28	-0.2	-0.1	0.0	-0.1	0.1	-0.2	0.3	0.1	0.0	-0.1	0.0	-0.2	0.3	0.1	-0.3	-0.2	0.0	-0.1	0.3	-0.3	-0.1	0.3	0.1	0.2	0.1	0.1	0.1	1.0	0.0	-0.1	0.1	0.2	-0.4	-0.3	0.1	0.0
29	0.2	-0.2	-0.2	-0.3	0.5	0.0	0.0	0.3	0.2	-0.4	-0.2	-0.1	0.0	0.4	0.2	-0.1	0.4	0.0	0.3	0.1	-0.1	0.3	0.0	0.4	0.3	0.5	0.6	0.0	1.0	0.4	0.8	0.4	0.1	0.2	-0.1	0.3
30	0.0	0.0	0.2	0.2	-0.2	0.2	-0.3	-0.3	-0.3	0.2	0.1	0.3	-0.2	0.4	0.1	0.0	-0.1	0.0	0.0	0.4	0.4	-0.2	-0.1	-0.1	-0.3	-0.2	-0.2	-0.1	0.4	1.0	-0.1	0.0	0.2	0.1	0.2	-0.2
31	0.2	-0.1	-0.4	-0.5	0.6	-0.1	0.1	0.5	0.4	-0.5	-0.3	-0.3	0.1	0.1	0.2	-0.1	0.4	0.0	0.3	-0.2	-0.3	0.4	0.0	0.4	0.5	0.7	0.7	0.1	0.8	-0.1	1.0	0.2	0.1	0.2	-0.3	0.5
32	-0.1	-0.3	0.1	0.0	0.3	0.1	0.0	0.0	-0.1	-0.2	-0.2	0.0	-0.1	0.1	-0.2	-0.2	0.2	-0.1	0.4	-0.1	-0.1	0.1	0.0	0.3	0.1	0.2	0.2	0.2	0.4	0.0	0.2	1.0	-0.2	-0.1	0.0	-0.1
33	0.9	0.2	-0.5	-0.4	0.2	0.4	-0.5	0.3	0.3	-0.2	-0.3	0.3	-0.5	0.3	1.0	0.5	0.5	0.1	-0.5	0.5	0.4	-0.4	-0.2	0.2	0.1	0.1	0.1	-0.4	0.1	0.2	0.1	-0.2	1.0	0.9	-0.3	0.5
34	0.9	0.1	-0.5	-0.4	0.4	0.4	-0.5	0.4	0.3	-0.4	-0.5	0.3	-0.5	0.2	1.0	0.4	0.6	0.1	-0.2	0.5	0.4	-0.3	-0.1	0.2	0.3	0.3	0.3	-0.3	0.2	0.1	0.2	-0.1	0.9	1.0	-0.3	0.5
35	-0.4	-0.1	0.5	0.4	-0.2	0.1	-0.2	-0.3	-0.3	0.2	0.3	0.1	0.3	-0.1	-0.3	-0.2	-0.3	0.0	0.1	-0.2	0.5	-0.2	0.1	-0.1	-0.3	-0.3	-0.3	0.1	-0.1	0.2	-0.3	0.0	-0.3	-0.3	1.0	-0.4
36	0.7	0.3	-0.8	-0.8	0.7	-0.4	0.3	0.8	0.8	-0.5	-0.2	-0.6	0.0	0.2	0.5	0.2	0.5	0.1	-0.1	-0.1	-0.2	0.4	-0.1	0.5	0.7	0.8	0.8	0.0	0.3	-0.2	0.5	-0.1	0.5	0.5	-0.4	1.0